## CONDIZIONAMENTO

**Es:** $\gamma \in (0,1)$, $A(\gamma) = \begin{bmatrix} \gamma & 0 \\ 0 & 1/\gamma \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$,

$\delta b = \begin{bmatrix} \delta b_1 \\ \delta b_2 \end{bmatrix}$ "perturbazione di b" $(\delta A = 0)$

- $\det A(\gamma) = 1$ (matr invert per ogni $\gamma$)
- soluz sist non perturbato $A(\gamma)x = b$: $x^* = \begin{bmatrix} b_1/\gamma \\ \gamma b_2 \end{bmatrix}$
- soluz sist perturbato $A(\gamma)x = b + \delta b$: $\hat{x} = \begin{bmatrix} (b_1+\delta b_1)/\gamma \\ \gamma(b_2+\delta b_2) \end{bmatrix}$
- 1° caso: $b_1 \neq 0$, $b_2 = 0$ $\Rightarrow \epsilon_d = \dfrac{\|\hat{x}-x^*\|_\infty}{\|x^*\|_\infty} = \dfrac{\left\| \begin{bmatrix} \delta b_1/\gamma \\ \gamma \delta b_2 \end{bmatrix} \right\|_\infty}{|b_1|}\gamma$

  $= \dfrac{\left\| \begin{bmatrix} \delta b_1 \\ \gamma^2 \delta b_2 \end{bmatrix} \right\|_\infty}{\|b\|_\infty} \leq \epsilon_b$ "ben condizionato" ($\forall$ perturbaz $\delta b$ ...)
- 2° caso: $b_1 = 0$, $b_2 \neq 0$; $\delta b_2 = 0$ $\Rightarrow \epsilon_d = \dfrac{|\delta b_1|}{\gamma} \cdot \dfrac{1}{\gamma |b_2|} = \dfrac{1}{\gamma^2}\epsilon_b$

  "mal condizionato" ($\exists$ perturbaz $\delta b$ ...)

---

## STUDIO delle F di CONDIZIONAM

$$\epsilon_d = F(A, b; \epsilon_A, \epsilon_b)$$

* CASO $\delta A = 0$ $\quad x^* = A^{-1}b$, $\hat{x} = A^{-1}(b+\delta b)$.

$\hat{x} - x^* = A^{-1}\delta b$; $Ax^* = b \Rightarrow \|b\| \leq \|A\|\,\|x^*\|$

$$\epsilon_d = \frac{\|\hat{x}-x^*\|}{\|x^*\|} = \frac{\|A^{-1}\delta b\|}{\|x^*\|} \leq \boxed{\|A^{-1}\|\,\|A\|}\,\epsilon_b$$

$\mu(A)$ "NUMERO di CONDIZIONAM di A" (in norma ...)

> **TEO 1:** $A \in \mathbb{R}^{n\times n}$;
>
> $\epsilon_d \leq \mu(A)\,\epsilon_b$ e $\exists\, b, \delta b \in \mathbb{R}^n$ t.c. $\epsilon_d = \mu(A)\,\epsilon_b$

**Es** (continua): $\bullet$ $\|A(\gamma)\|_\infty = \frac{1}{\gamma}$, $\|A^{-1}\|_\infty = \frac{1}{\gamma}$

$\bullet$ $\mu_\infty(A) = \frac{1}{\gamma^2} \Rightarrow \epsilon_d \leq \frac{1}{\gamma^2}\epsilon_b$; $b = \begin{bmatrix} 0 \\ b_2 \end{bmatrix}$, $\delta b = \begin{bmatrix} \delta b_1 \\ 0 \end{bmatrix}$ ...

* CASO $\delta b = 0$

> **TEO 2:** $A \in \mathbb{R}^{n\times n}$
>
> $\hat{\epsilon}_d \leq \mu(A)\,\epsilon_A$ e $\exists\, b, \delta A \in \mathbb{R}^{n\times n}$ t.c. $\hat{\epsilon}_d = \mu(A)\,\epsilon_A$

**Oss:** se $\mu(A) \gg 1$, $\exists\, b \in \mathbb{R}^n$, $\delta b \in \mathbb{R}^n$, $\delta A \in \mathbb{R}^{n\times n}$ t.c.

$\epsilon_d$ oppure $\hat{\epsilon}_d \gg \epsilon_b, \epsilon_A$ ("mal condizionato")

---

**Oss:** $A \in \mathbb{R}^{n\times n}$; $\mu(A) \geq 1$ per ogni norma

**Oss:** $A \in \mathbb{R}^{n\times n}$; $\mu(A) = \dfrac{\max\{N(Av),\ N(v)=1\}}{\min\{N(Av),\ N(v)=1\}}$

**Es:** $A = \begin{bmatrix} 3 & 4 & 0 \\ 1 & -1 & -3 \\ 0 & 7 & 1 \end{bmatrix}$
- Calcolare $\|A\|_1$;
- Calcolare $\|Ae_k\|_1$ per $k=1,2,3$; e dedurne una stima per $\mu_1(A)$;
- stimare $\|A\|_2$ e $\mu_2(A)$.

**Es:** $T \in \mathbb{R}^{n\times n}$ tr sup; verif che $\|T\|_1 \geq \max_k |t_{kk}|$

e, se T inv, che $\|T^{-1}\|_1 \geq \max_k |t_{kk}^{-1}|$.

**Es:** $\gamma \in (0,1)$, $A(\gamma) = \begin{bmatrix} \gamma & 1 \\ 1 & 0 \end{bmatrix}$;
- determ fatt LR con EG (e con EGP ?)
- discutere condiz del fatt destro.