Circuits of Resistors, Diodes and Sources: Topological Conditions Sufficient to Define the State of a Diode

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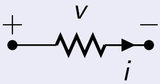
Good afternoon everybody.

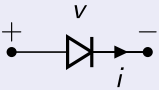
In this presentation we consider circuits of resistor diodes and sources.

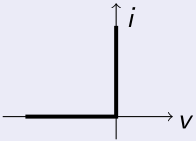
RDS circuits:

- *positive* linear **R**esistors

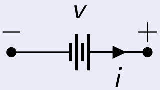


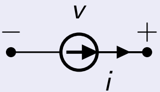
- *ideal* **D**iodes





- *positive* independent voltage and current **S**ources



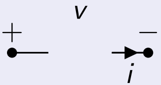


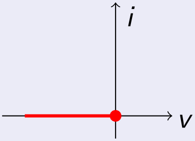
More precisely we consider circuits composed of a finite number of positive linear resistors, ideal diodes and independent sources of positive values, and we call them RDS circuits.

As usual, by an ideal diode we mean a diode whose characteristic is that represented in the picture.

State of a diode, given an operating point:

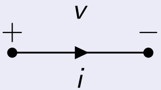
- open

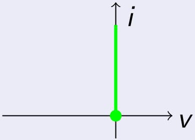




$$i = 0$$
$$v \leqslant 0$$

- closed



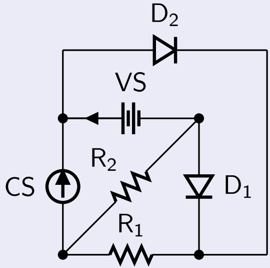


$$i \geqslant 0$$
$$v = 0$$

We will say that a diode is in the open state if its operating point has zero current, and that it is in the closed state if its operating point has zero voltage.

Usual analysis problem:

Given a RDS circuit N, for example:



find *all* the solutions of N.

Given a RDS circuit, for example the one represented in the picture, a usual analysis problem is to find all the solutions of the circuit.

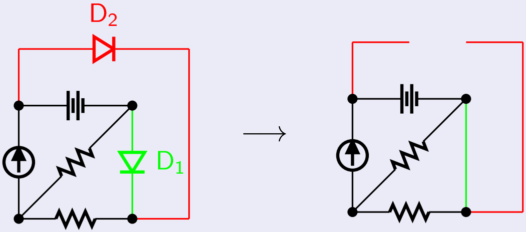
Usual elementary procedure:

Let d be the number of diodes of N.

for each of the 2^d combinations of states of the diodes, **repeat**:

1. **replace** all the open diodes by open circuits, all the closed diodes by short circuits, **and find all the solutions** of the *linear* circuit obtained
2. among these solutions, **find those consistent** with the state of the diodes under consideration — each of these is a solution of N

end



A usual elementary and general procedure to find all the solutions of the circuit is the following.

For each of the 2^d combinations of states of the diodes (for example: D_1 closed, D_2 open), repeat these two steps:

step one: replace all the open diodes by open circuits, all the closed diodes by short circuits, and find all the solutions of the linear circuit obtained.

step two: among all the solutions found in step one, find those consistent with the state of the diodes under consideration (each of these is a solution of N).

Drawback of the elementary procedure:

we must solve 2^d linear circuits

Our aim:

To find a **simple and rigorous** technique to **predict** (i.e.: to determine without any numerical computation) the state of some of the diodes of N, so to **significantly reduce** the number of linear circuits to be solved.

Known results:

- Belevitch (1962): no prediction; Vandewalle and Chua (1980): simple but not general; Hasler et al. (1989): general but not simple.

This procedure has the drawback that we must solve 2^d linear circuits.

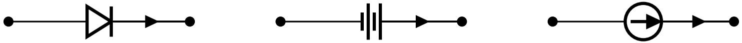
Our aim is to find a simple and rigorous technique to predict, that is to determine without any numerical computation, the state of some of the diodes of the circuit. This will significantly reduce the number of circuits to be solved.

Some results are known: Belevitch has found an upper bound to the number of combinations of diode states to be considered in the elementary procedure but he has given no technique to identify which combinations should be considered. Vandewalle and Chua have found a simple technique to predict the state of some diodes, but their results can be applied only if the circuit has a very particular structure. Finally, Hasler and his co-workers have found a general technique to identify combinations of diode states not to be considered in the elementary procedure, but it is very complex.

We will state three pairs of conditions. Each pair will have a condition, which we label with C, sufficient to predict the closed state of a diode, and one, which we label with O, to predict its open state.

orientation of diodes and sources

We introduce an orientation for each diode and source of N:



In our technique, only the reciprocal orientations of the diodes and sources will play a role – we may forget the actual (positive) values of the resistors and sources.

To state our conditions, we introduce an orientation for each of the diodes and sources in the circuit. As in the picture: each diode is oriented in the forward direction, each positive c.s. is oriented in the direction of the current flow and each v.s. is oriented from the negative to the positive terminal.

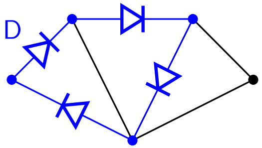
In our technique, only the reciprocal orientations of the diodes and sources will play a role – in particular, we may forget the actual (positive) values of the resistors and sources.

first result: conditions (C_1) and (O_1)

Let D be one of the diodes of N.

- **If** D is part of a loop of equi-oriented diodes, **then** for every solution of N, D is in the closed state.

- **If** D is part of a cut-set of equi-oriented diodes, **then** for every solution of N, D is in the open state.



$$\sum_{\text{loop}} v_k = 0 \quad \& \quad \forall k, v_k \leqslant 0$$

$$\Downarrow$$

$$\forall k, v_k = 0$$

Here is our first pair of conditions.

Let D be one of the diodes of our circuit.

First statement: If D is part of a loop of equi-oriented diodes, that is: it is part of a loop of diodes in which all the diodes have the same orientation in the loop (for example the blue loop on the left of the picture where all the diodes are clockwise oriented), then we can predict the state of the diode D to be closed.

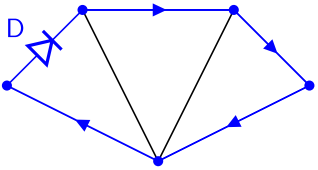
The straightforward proof of the statement is shown on the right of the picture.

The second statement is the dual of the previous one: If D is part of a cut-set of equi-oriented diodes, then we can predict the state of the diode D to be open.

second result: intro to (O_2)

Let D be **not** part of a loop of equi-oriented diodes.

- **If** there exists a solution of N in which the current in D is positive **then** in N there exists a loop ℓ containing D and in all whose branches the current is nonzero and flows in the same direction (the proof uses the Colored Branch Theorem).



(1) D′ ≠ D in $\ell \Rightarrow$ D′ and D have the same orientation

(2) KVL $\Rightarrow$ at least one source is in ℓ

(3) c.s. in $\ell \Rightarrow$ c.s. and D have the same orientation

(4) KVL $\Rightarrow$ if no c.s. in ℓ then at least one of the v.s. in ℓ must be oriented as D.

Before we state our next condition, let us sketch its two-step proof.

Let D be not part of a loop of equi-oriented diodes.

First step: if there exists a solution in which the current in D is positive, then there exists a loop containing D and in all whose branches the current is nonzero and flows in the same direction, for example the blue loop in the picture.

Second step: we ask now which elements may occupy the anonymous branches of the loop. We get four properties.

First one: if the loop contains other diodes, then they must be oriented as D.
Second one: by the KVL, the loop must contain at least one source.
Third one: if the loop contains some (positive) current sources, then they must be oriented as D.
Last one: by the KVL again, if in the loop there are no current sources, then at least one of the (positive) voltage sources in the loop must be oriented as D.

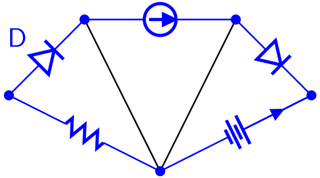
This statement gives a necessary condition to the existence of a solution with a positive current in D. We get a sufficient condition to predict the open state of D by reformulating the statement...

second result: condition (O_2)

If D is **not** part of a loop of equi-oriented diodes, we have:

- **if** there does **not** exist a loop ℓ containing D and such that
  - (1) all the diodes in ℓ have the same orientation
  - (2) ℓ contains at least one independent source
  - (3) if ℓ contains some c.s., then they must be oriented as D
  - (4) if no c.s. are in ℓ, then there exists a v.s. in ℓ which has the same orientation as D

  then for every solution of N, the current in D is not positive, i.e. D is in the open state.



...in this equivalent way.

If there does not exist a loop containing D and which has the previous four properties, then we can predict the state of the diode D to be open. This is condition (O_2).

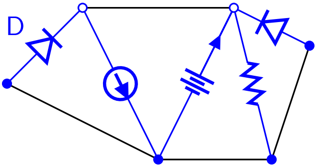
In the picture we see two examples of loops which have all the four properties.

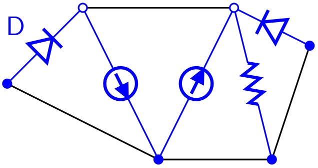
second result: conditions (C_2)

If D is **not** part of a cut-set of equi-oriented diodes, we have:

- **if** there does **not** exist a cut-set k containing D and such that
  - (1*) all the diodes in k have the same orientation
  - (2*) k contains at least one independent source
  - (3*) if k contains some v.s., then they must be oriented as D
  - (4*) if no v.s. are in k, then there exists a c.s. in k which has the same orientation as D

  then for every solution of N, D is in the closed state.





Condition (C_2) is the dual of (O_2). We give directly the final formulation.

Let D be not part of a cut-set of equi-oriented diodes. We have: if there does not exist a cut-set containing D and which has the four properties dual of the previous ones, then we can predict the state of the diode D to be closed.

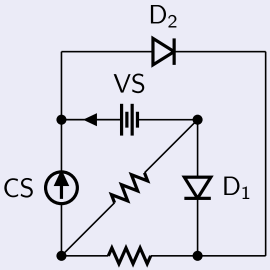
In the picture we see two examples of cut-sets which have all the four properties.

So far we have shown two pairs of conditions. A first one very easy to check and a second one not so easy to check as the first one. Our third and last pair of conditions is very intuitive, and simpler to check but less powerful than the second one. To state it we need a definition.

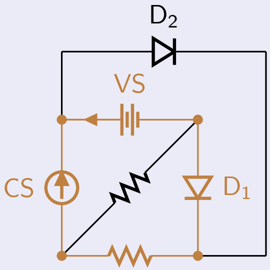
totally equi/anti-oriented diode

Definition

A diode D of *N* is totally equi-oriented [totally anti-oriented] if: for **every** source S of *N*, the diode D and the source S have the same [the opposite] orientation in **any** loop containing both elements.



D_2 is totally equi-oriented



D_1 is not totally oriented

We say that a diode is totally equi-oriented if for every source in the circuit the diode and the source have the same orientation in any loop containing both elements. The diode is totally anti-oriented if for every source in the circuit the diode and the source have the opposite orientation in any loop containing both elements.

Observe that the definition uses only the notion of loop.

Practically, a diode is totally equi-oriented when all the sources push the current in the forward direction of the diode, and it is totally anti-oriented when all the sources push the current in the reverse direction.

In the picture: D_2 is totally equi-oriented, and D_1 is neither totally equi-oriented nor totally anti-oriented. Indeed, in the brown loop D_1 and VS have the opposite orientation and D_1 and CS have the same orientation.

main results: conditions (C_3) and (O_3)

If D is **not** part of a cut-set of equi-oriented diodes, we have:

- **if** D is totally equi-oriented **then** for every solution of N, D is in the closed state.

If D is **not** part of a loop of equi-oriented diodes, we have:

- **if** D is totally anti-oriented **then** for every solution of N, D is in the open state.

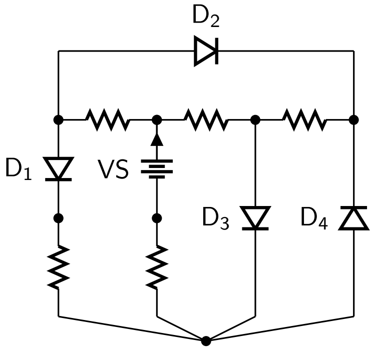
Here is our last pair of conditions.

First: Let assume that D is not part of a cut-set of equi-oriented diodes. We have: if D is totally equi-oriented then we can predict the state of the diode D to be closed.

Second: Let assume that D is not part of a loop of equi-oriented diodes. In this case we have: if D is totally anti-oriented then we can predict the state of the diode D to be open.

These conditions agree with our intuition: if all the sources push the current in the forward direction of the diode then its state will be closed, and if all the sources push the current in the reverse direction of the diode, then its state will be open.

example



- No diode-only loops and no diode-only cut-sets
- D_1 and D_3 are totally equi-oriented: $(C_3) \Rightarrow$ closed
- D_4 is totally anti-oriented: $(O_3) \Rightarrow$ open
- D_2 is not totally oriented but it verifies condition $(C_2) \Rightarrow$ closed.

Finally, an example of application of our conditions.

Consider the circuit in the picture.

It is immediately seen that: there are neither loops nor cut-sets of equi-oriented diodes; that D_1 and D_3 are totally equi-oriented, so they satisfy condition (C_3) and we can predict their state to be closed, and that D_4 is totally anti-oriented, so it satisfies condition (O_3) and we can predict its state to be open.

Also immediate is that D_2 is not totally oriented: neither (C_3) nor (O_3) can be used to predict its state.

With a bit of patience we can prove that it verifies condition (C_2) and so we can predict its state to be closed.

conclusions

- We presented **six topological conditions**, each one **sufficient to predict the state of a diode** in an RDS circuit.
- If D verifies one whatever of the conditions, then the state of D is **known and independent of the values of the resistors and sources**.
- The conditions are **simple and effective enough** to predict, sometimes by inspection, the state of some diodes of many small circuits. On the other hand, it seems to be hard to check them in large circuits.
- The first pair of conditions can be applied to **circuits containing also linear active elements**, and the second pair can be generalized to such class of circuits.

Last slide: concluding remarks.

Thank you for your attention.