CAFG $\boxed{26}$ / 16 aprile 2010 / A22

<u>Es</u>: $A = \begin{bmatrix} 2 & -1 & -2 & 3 \\ 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \in \mathbb{C}^{4\times 4}$; det FCJ(A) e matr che realizza la sim.

- $p_A(x) = (-x)(1-x)\left[(2-x)(-x)+1\right] = (-x)(1-x)(x^2-2x+1)$

  $= (-x)(1-x)^3$

- $\exists$ blocchi associati a 0, Σ dim $= 1 \Rightarrow J_1(0)$

  $\exists$ blocchi associati a 1, Σ dim $= 3$

- $\dim \ker(A-I) = \dim \ker \begin{bmatrix} 1 & -1 & -2 & 3 \\ 1 & -1 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = 1$

  $\Rightarrow$ # blocchi assoc a $1 = 1 \Rightarrow J_3(1)$

- FCJ(A) = diag$(J_1(0), J_3(1))$ = $\begin{bmatrix} 0 & & & \\ & 1 & 1 & \\ & & 1 & 1 \\ & & & 1 \end{bmatrix}$

- Cerchiamo $c_1, \dots, c_4$ lin indip t.c. $C = (c_1 \dots c_4)$ verifica: $AC = C\,$FCJ(A)

- Per colonne:

  1) $Ac_1 = 0$
  2) $Ac_2 = c_2$
  3) $Ac_3 = c_2 + c_3$
  4) $Ac_4 = c_3 + c_4$

  1) $c_1 \begin{cases} \in \ker A \\ \neq 0 \end{cases}$

  2) $c_2 \begin{cases} \in \ker(A-I) \\ \neq 0 \end{cases}$

  3) $c_3 \begin{cases} \notin \ker(A-I) & [(A-I)c_3 = c_2 \neq 0] \\ \in \ker(A-I)^2 & [(A-I)^2 c_3 = \dots = 0] \end{cases}$

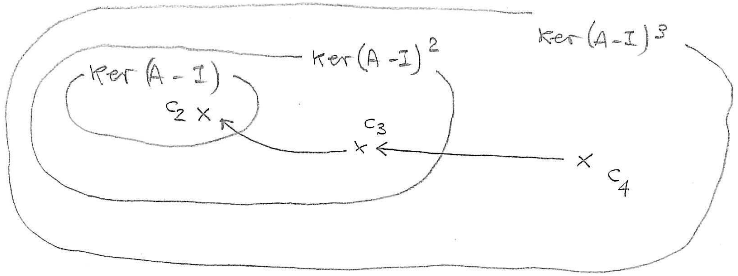
  4) $c_4 \begin{cases} \notin \ker(A-I) & [(A-I)c_4 = c_3 \neq 0] \\ \notin \ker(A-I)^2 & [(A-I)^2 c_4 = (A-I)c_3 = c_2 \neq 0] \\ \in \ker(A-I)^3 & [(A-I)^3 c_4 = (A-I)c_2 = 0] \end{cases}$

- $\dim \ker A = 1$

  $\dim \ker(A-I) = 1$

  $\dim \ker(A-I)^2 = 2$ e $\ker(A-I)^2 \supset \ker(A-I)$

  $\dim \ker(A-I)^3 = 3$ e $\ker(A-I)^3 \supset \ker(A-I)^2$



- $c_1 \begin{cases} \neq 0 \\ \in \ker A \end{cases}$ (esiste !)

- $c_4 \begin{cases} \in \ker(A-I)^3 \\ \notin \ker(A-I)^2 \end{cases}$ (esiste perché ...)

  $\Rightarrow c_3 = (A-I)c_4 \begin{cases} \notin \ker(A-I) \\ \in \ker(A-I)^2 \end{cases}$

  $\Rightarrow c_2 = (A-I)c_3 \begin{cases} \in \ker(A-I) \\ \neq 0 \end{cases}$

- $\ker A = \left\langle \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle$

  $\ker(A-I) = \ker \begin{bmatrix} 1 & -1 & -2 & 3 \\ 1 & -1 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \left\langle \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \right\rangle$

  $\ker(A-I)^2 = \ker \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \left\langle \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\rangle$

  $\ker(A-I)^3 = \ker \begin{bmatrix} 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \left\langle \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\rangle$

- $c_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$; $c_4 = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \Rightarrow c_3 = (A-I)c_4 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \Rightarrow c_2 = (A-I)c_3 = \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

  $\Rightarrow C = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$, $\det C = -1 \; (\neq 0\,!)$