Riassunto:

- matrice in f. di Jordan "determinata" dall'elenco dei blocchi sulla diagonale (ordine escluso!)
- info suff per determ l'elenco per A in f di J:
  1) polinomio caratt di A;
  2) $\forall \lambda$ autovalore e k intero ≥ 0: $\dim \operatorname{Ker} (A-\lambda I)^k$.

Oss: $A \in \mathbb{C}^{n \times n}$;

- Teo: $\exists$ FCJ(A)
- polin caratt e $\dim \operatorname{Ker} (A-\lambda I)^k$ invar per similit

$\Rightarrow$ è poss determ FCJ(A) direttam da A.

Es: $A \in \mathbb{C}^{5\times 5}$ t.c.

(1) $p_A(x) = (1-x)^3 (i-x)^2$

(2) $\dim \operatorname{Ker}(A-I) = 2$, $\dim \operatorname{Ker}(A-iI) = 2$

Determ, se poss, FCJ(A).

Sol: (1) $\Rightarrow$
- $\exists$ blocchi associati a 1, Σ dim $= 3$
- $\exists$ blocchi associati a i, Σ dim $= 2$

(2) $\Rightarrow$
- # blocchi associati a 1: 2 $\Rightarrow$ $J_1(1)$, $J_2(1)$
- # blocchi associati a i: 2 $\Rightarrow$ $J_1(i)$, $J_1(i)$

q.di: $\text{FCJ}(A) = \operatorname{diag}\big(J_1(1), J_2(1), J_1(i), J_1(i)\big)$

Pb (per casa):

- $A \in \mathbb{C}^{5\times 5}$ t.c. (1) $p_A(x) = (1-x)^3 (i-x)^2$
  (2) $\dim \operatorname{Ker}(A-I) = 3$, $\dim \operatorname{Ker}(A-iI) = 2$

determ, se poss, FCJ(A).

- $A = \operatorname{diag}\big(J_2(1), J_2(2), J_1(2)\big) \in \mathbb{C}^{5\times 5}$

determ $p_A(x)$ e, per ciascun autovalore λ: $\dim \operatorname{Ker}(A-\lambda I)$ e $\dim \operatorname{Ker}(A-\lambda I)^2$.

Es: $A = \begin{bmatrix} 2 & -1 & -2 & 2 \\ 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \in \mathbb{C}^{4\times 4}$; determ FCJ$(A)$.

- $p_A(x) = (1-x)(-x)\left[(2-x)(-x)+1\right] = (1-x)^3(-x)$

$\Rightarrow$ $\exists$ blocchi associati a 1 e Σ dim $= 3$
$\exists$ blocchi associati a 0 e Σ dim $= 1$ $\Rightarrow$ $J_1(0)$

- $\dim \operatorname{Ker}(A-I) = \dim \operatorname{Ker} \begin{bmatrix} 1 & -1 & -2 & 2 \\ 1 & -1 & -1 & 1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = 2$

$\Rightarrow$ # blocchi associati a 1: due $\Rightarrow$ $J_1(1)$, $J_2(1)$.

dunque: $\text{FCJ}(A) = \operatorname{diag}\big(J_1(0), J_1(1), J_2(1)\big)$.

Pb (per casa): $A = \begin{bmatrix} 2 & 0 & -1 \\ 1 & 1 & -2 \\ 0 & 0 & 2 \end{bmatrix} \in \mathbb{C}^{3\times 3}$; determ FCJ$(A)$.

Es: $A = \begin{bmatrix} 2 & 0 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{bmatrix} \in \mathbb{C}^{4\times 4}$; determ FCJ$(A)$.

- $p_A(x) = (2-x)^4$ $\Rightarrow$ $\exists$ blocchi associati a 2 e Σ dim $= 4$
- $\dim \operatorname{Ker}(A-2I) = 2$ $\Rightarrow$ # blocchi associati a 2: due
- $\dim \operatorname{Ker}(A-2I)^2 = 3$ $\Rightarrow$ # blocchi dim ≥ 2: uno

$\Rightarrow$ $J_1(2)$, $J_3(2)$

dunque: $\text{FCJ}(A) = \operatorname{diag}\big(J_1(2), J_3(2)\big)$

Pb (per casa): det tutte le matr in f di Jordan simili alla matr A dei tre es precedenti

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Pb: data $A \in \mathbb{C}^{n\times n}$ e $FCJ(A)$,
determ $C \in \mathbb{C}^{n\times n}$ che realizza la similitudine.

[ovvero: C invertibile e $AC = C\, FCJ(A)$]

Es: $A = \begin{bmatrix} 2 & -1 & -2 & 2 \\ 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \in \mathbb{C}^{4\times 4}$, $FCJ(A) = \mathrm{diag}(J_1(0), J_1(1), J_2(1))$

- cerchiamo $c_1, c_2, c_3, c_4 \in \mathbb{C}^4$ lin indip t.c.

$$A(c_1, c_2, c_3, c_4) = (c_1, c_2, c_3, c_4)\, FCJ(A)$$

- per colonne:
  1) $Ac_1 = 0$
  2) $Ac_2 = c_2$
  3) $Ac_3 = c_3$
  4) $Ac_4 = c_3 + c_4$ ovvero $(A-I)c_4 = c_3$

- dunque:

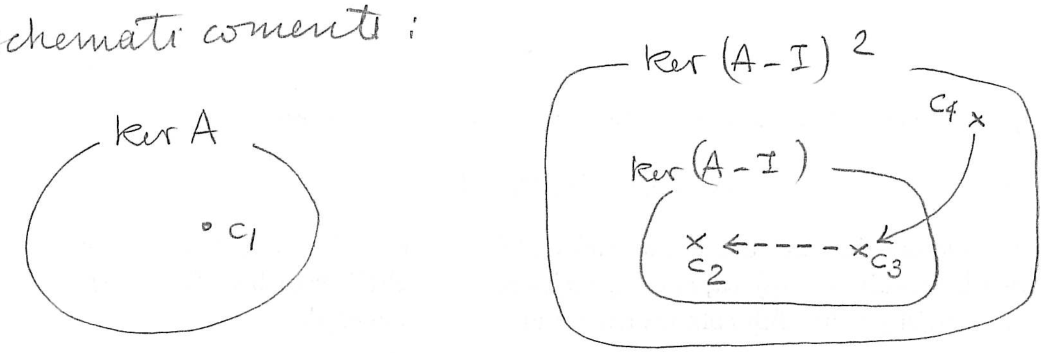
(1) $c_1 < \begin{matrix} \neq 0 \\ \in \ker A \end{matrix}$ | (2) $c_2 < \begin{matrix} \neq 0 \\ \in \ker(A-I) \end{matrix}$

(3) $c_3 < \begin{matrix} \neq 0 \\ \in \ker(A-I) \end{matrix}$ | (4) $(A-I)c_4 = c_3 \neq 0 \Rightarrow c_4 \notin \ker(A-I)$
$(A-I)^2 c_4 = (A-I)c_3 = 0 \Rightarrow c_4 \in \ker(A-I)^2$

- $\dim \ker A = 1$
  $\dim \ker(A-I) = 2$, $\dim \ker(A-I)^2 = 3$
  e $\ker(A-I)^2 \supset \ker(A-I)$

- (1) c_1 = un quals elem non nullo di $\ker A$
  (2) c_4 = un quals elem di $\ker(A-I)^2$ non appartenente a $\ker(A-I)$
  (3) $c_3 = (A-I)c_4$ [$\Rightarrow c_3 \in \ker(A-I)$ e $c_3 \neq 0$]
  (4) c_2 = un elem di $\ker(A-I)$ lin indip da c_3.

- schemati commenti:



- $\ker A = \left\langle \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \right\rangle$

$\ker(A-I) = \left\langle \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\rangle$, $\ker(A-I)^2 = \left\langle \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \right\rangle$

- $c_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$, $c_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$

$c_3 = (A-I)c_4 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$, $c_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$

- $C = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ $(\det C = -1 \; (\neq 0))$