Oss: $A = \overset{\in\mathbb{C}^{m\times m}}{U}\,\underset{\in\mathbb{C}^{m\times k}}{\Sigma}\,\underset{\in\mathbb{C}^{k\times k}}{V^H}$;

- $x \in \ker A \iff \underset{\overset{\parallel}{x'}}{V^H x} \in \ker \Sigma$ (x' coord x risp $v_1, \dots, v_k$)

$\Rightarrow \boxed{\dim\ker A = \dim\ker\Sigma}$

- teo dim $\Rightarrow \boxed{rk(A) = rk(\Sigma)} = \#$ valori singolari non nulli $= r$
- $AV = (Av_1, \dots, Av_r, Av_{r+1}, \dots, Av_k) = U\Sigma = (\sigma_1 u_1, \dots, \sigma_r u_r, 0, \dots, 0)$

$$\Rightarrow \begin{cases} \ker A = \langle \underbrace{v_{r+1}, \dots, v_k}_{\text{base o.n}} \rangle & (\dim\ker A = k-r) \\ \operatorname{im} A = \langle \underbrace{u_1, \dots, u_r}_{\text{base on}} \rangle & (\dim \operatorname{im} A = r) \end{cases}$$

Es: $A = (u_1, u_2)\begin{pmatrix} 2 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}(v_1, v_2, v_3)^H$;

- $rk(A) = 1$, $\operatorname{im}(A) = \langle u_1 \rangle$ | $\operatorname{im}(\Sigma) = \langle \begin{pmatrix} 1 \\ 0 \end{pmatrix} \rangle$
- $\dim\ker(A) = 2$, $\ker A = \langle v_2, v_3 \rangle$ | $\ker(\Sigma) = \langle \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \rangle$

Oss: $A \in \mathbb{C}^{m\times m}$ hermitiana; $Q \in \mathbb{C}^{m\times m}$ unitaria, $\lambda_1, \dots, \lambda_m \in \mathbb{R}$ t.c. $|\lambda_1| \geq \dots \geq |\lambda_m|$, $A = Q \operatorname{diag}(\lambda_1, \dots, \lambda_m) Q^H$.

Allora: posto $s_j = \begin{cases} 1 & \text{se } \lambda_j \geq 0 \\ -1 & \text{se } \lambda_j < 0 \end{cases}$ e $W = (s_1 q_1, \dots, s_m q_m)$ si ha

- $A = Q \operatorname{diag}(|\lambda_1|, \dots, |\lambda_m|) W^H$
- $W^H W = I$

q. di $Q, \operatorname{diag}(|\lambda_1|, \dots, |\lambda_m|), W$ e' decomp ai v s di A.

Es: $A = (q_1, q_2, q_3)\begin{pmatrix} -3 & & \\ & 1 & \\ & & -1 \end{pmatrix}(q_1, q_2, q_3)^H = (q_1, q_2, q_3)\begin{pmatrix} 3 & & \\ & 1 & \\ & & 1 \end{pmatrix}\underbrace{\begin{pmatrix} -1 & & \\ & 1 & \\ & & -1 \end{pmatrix}(q_1, q_2, q_3)^H}_{(-q_1, q_2, -q_3)^H}$

Oss: $A = U\Sigma V^H \in \mathbb{C}^{m\times m}$;

- $M \in \mathbb{C}^{m\times m}$ unitaria $\Rightarrow |\det M| = 1$;
- $|\det A| = |\det U|\,|\det\Sigma|\,|\det V^H| = \det\Sigma = \sigma_1 \cdots \sigma_m$;
- A invertibile $\iff$ tutti i v. sing di A sono positivi.