Es: $A \in \mathbb{C}^{2\times 2}$, $FCJ(A) = \begin{bmatrix}\lambda & 1\\ 0 & \lambda\end{bmatrix}$, matr che realizza la sim: $C = (c_1, c_2)$

- $\hat{v} = C^{-1}v$ (vett delle coord di v risp alla base c_1, c_2)

  $\hat{x}_k = C^{-1}x_k$

- $x_k = A^k v \Rightarrow \hat{x}_k = C^{-1}x_k = C^{-1}A^k v = C^{-1}A^k C\hat{v} = (FCJ(A))^k \hat{v}$

  quindi: $\hat{x}_k = \begin{bmatrix}\lambda^k & k\lambda^{k-1}\\ 0 & \lambda^k\end{bmatrix}\hat{v}$ (vedere Es precedente)

  e $x_k = C\hat{x}_k$ (x_k limitata $\Leftrightarrow$ $\hat{x}_k$ limitata, x_k convergente $\Leftrightarrow$ $\hat{x}_k$ convergente)

Es: $A = \begin{bmatrix}3&2&0&0\\0&3&0&0\\-1&0&3&2\\0&1&0&3\end{bmatrix}$; $FCJ(A) = \begin{bmatrix}3&1&&\\&3&&\\&&3&1\\&&&3\end{bmatrix}$, $C = \begin{bmatrix}0&1&2&0\\0&0&0&1\\-1&0&0&0\\0&0&1&0\end{bmatrix}$

- $x_k = A^k v$, $\hat{x}_k = (FCJ(A))^k\hat{v} = \begin{bmatrix}3^k&k3^{k-1}&&\\&3^k&&\\&&3^k&k3^{k-1}\\&&&3^k\end{bmatrix}\hat{v}$, $x_k = C\hat{x}_k$

$v = \begin{bmatrix}1\\1\\0\\0\end{bmatrix}$, $\hat{v} = \begin{bmatrix}0\\1\\0\\1\end{bmatrix}$; $\hat{x}_k = \begin{bmatrix}k3^{k-1}\\3^k\\k3^{k-1}\\3^k\end{bmatrix}$, $x_k = \begin{bmatrix}3^k+2k3^{k-1}\\3^k\\-k3^{k-1}\\k3^{k-1}\end{bmatrix}$ $(k \geq 1)$

- $\hat{v}_3 = \hat{v}_4 = 0 \Rightarrow \hat{x}_{k,3} = \hat{x}_{k,4} = 0$ per $k = 1, 2, \ldots$

  $\hat{v}_1 = \hat{v}_2 = 0 \Rightarrow \hat{x}_{k,1} = \hat{x}_{k,2} = 0$ per $k = 1, 2, \ldots$

- $S_{12} = \langle c_1, c_2\rangle$, $S_{34} = \langle c_3, c_4\rangle$; $v \in S_{12} \Rightarrow x_k \in S_{12}$ per $k\ldots$

  $v \in S_{34} \Rightarrow x_k \in S_{34}$ per $k\ldots$

- $\underbrace{x \in S_{12}}_{\Updownarrow\ \hat{x}_3 = 0,\ \hat{x}_4 = 0} \Rightarrow Ax \in S_{12}$ $\left(x \in S_{12} \Rightarrow \begin{matrix}\hat{x}_3 = 0\\ \hat{x}_4 = 0\end{matrix} \Rightarrow \begin{matrix}(FCJ(A)\,\hat{x})_3 = 0\\ (FCJ(A)\,\hat{x})_4 = 0\end{matrix} \Rightarrow \begin{matrix}(\widehat{Ax})_3 = 0\\ (\widehat{Ax})_4 = 0\end{matrix} \Rightarrow Ax \in S_{12}\right)$

  $x \in S_{34} \Rightarrow Ax \in S_{34}$

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def: V sp vett, $\mathcal{L}: V \to V$ appl lin, S sottosp vett di V;

S sottosp invariante di $\mathcal{L}$ $\Leftrightarrow$ $\forall v \in S$, $\mathcal{L}(v) \in S$

- S_{12} e S_{34} sono sottosp invarianti dell'appl def da A.
- $\mathbb{C}^4 = S_{12} \oplus S_{34}$ $(S_{12} + S_{34} = \mathbb{C}^4,\ S_{12} \cap S_{34} = \{0\})$.

Es: $A = \begin{bmatrix}2&-1&-2&2\\1&0&-1&1\\0&0&0&1\\0&0&0&1\end{bmatrix}$, $FCJ(A) = \begin{bmatrix}0&&&\\&1&&\\&&1&1\\&&&1\end{bmatrix}$, $C = \begin{bmatrix}1&0&1&1\\0&0&1&0\\1&1&0&0\\0&1&0&0\end{bmatrix}$

- $S_1 = \langle c_1\rangle$, $S_2 = \langle c_2\rangle$, $S_{34} = \langle c_3, c_4\rangle$ sono sottosp invarianti
- $\mathbb{C}^4 = S_1 \oplus S_2 \oplus S_{34}$
- $v = \underbrace{v_1}_{\alpha_1 c_1} + \underbrace{v_2}_{\alpha_2 c_2} + \underbrace{v_{34}}_{\alpha_3 c_3 + \alpha_4 c_4}$

  $x_k = A^k v = A^k v_1 + A^k v_2 + A^k v_{34}$

  $A^k v_1 \to 0^k v_1 \in S_1$ (v_1 è autovett...)

  $A^k v_2 \to 1^k v_2 \in S_2$ (v_2 è autovett...)

  $A^k v_{34} \to$ coord $= \begin{bmatrix}0\\0\\\alpha_3 1^k + \alpha_4 k 1^{k-1}\\\alpha_4 1^k\end{bmatrix} \to (\alpha_3 1^k + \alpha_4 k 1^{k-1})c_3 + \alpha_4 1^k c_4 \in S_{34}$ (v_{34} NON è autovett...)

Oss: A, $FCJ(A)$ le stesse, $C' = \begin{bmatrix}1&1&1&2\\0&1&1&1\\1&1&0&1\\0&1&0&1\end{bmatrix}$;

- $\begin{cases}S'_1 = \langle c'_1\rangle,\ S'_2 = \langle c'_2\rangle,\ S'_{34} = \langle c'_3, c'_4\rangle\\ \mathbb{C}^4 = S'_1 \oplus S'_2 \oplus S'_{34}\end{cases}$ MA $S'_1 = S_1$, $S'_2 \neq S_2$, $S'_{34} \neq S_{34}$

  ovvero: scompos "NON CANONICA"

- invece È canonica:

  $\mathbb{C}^4 = S_1 \oplus S_{234} = \langle c_1\rangle \oplus \langle c_2, c_3, c_4\rangle = \langle c'_1\rangle \oplus \langle c'_2, c'_3, c'_4\rangle = S'_1 \oplus S'_{234}$

  (**Es:** verificare!) infatti: $S_1 = \ker(A)$, $S_{234} = \ker(A-I)^2$

Oss (scompos canonica):

$A \in \mathbb{C}^{n \times n}$, spettro di A: $\{\lambda_1, \dots, \lambda_k\}$, ν_j molt geom λ_j,

$\underbrace{\delta_{11}, \dots, \delta_{1\nu_1}}_{\max = n_1}; \dots; \underbrace{\delta_{k1}, \dots, \delta_{k\nu_k}}_{\max = n_k}$ dimens blocchi

Allora: $\boxed{\mathbb{C}^n = \ker(A - \lambda_1 I)^{n_1} \oplus \dots \oplus \ker(A - \lambda_k I)^{n_k}}$

Es: $FCJ(A) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 2 & 1 \\ & & & 2 \end{pmatrix}$; $\mathbb{C}^4 = \ker(A - I) \oplus \ker(A - 2I)^2$

$FCJ(A) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$; $\mathbb{C}^4 = \ker(A - I)$

$FCJ(A) = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 2 & \\ & & & 3 \end{pmatrix}$; $\mathbb{C}^4 = \ker(A - I) \oplus \ker(A - 2I) \oplus \ker(A - 3I)$