Pb: data A e $FCJ(A) \in \mathbb{C}^{n\times n}$, determinare $C \in \mathbb{C}^{n\times n}$ invert t.c. $AC = C\,FCJ(A)$.

Es: $A = \begin{bmatrix} 2 & -1 & -2 & 2 \\ 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \in \mathbb{C}^{4\times 4}$, $FCJ(A) = \begin{bmatrix} 0 & & & \\ & 1 & & \\ & & 1 & 1 \\ & & & 1 \end{bmatrix}$ (calcolata in es precedenti).

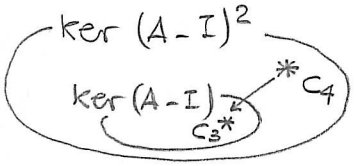
Cerco $c_1, c_2, c_3, c_4 \in \mathbb{C}^4$ lin indip t.c.

posto $C = (c_1, c_2, c_3, c_4) \in \mathbb{C}^{4\times 4}$ si abbia $AC = C\,FCJ(A)$

ovvero (leggendo l'uguaglianza per colonne):

- $Ac_1 = 0 \iff c_1 \in \ker(A)$
- $Ac_2 = c_2 \iff c_2 \in \ker(A-I)$
- $Ac_3 = c_3$ | $(A-I)c_3 = 0 \iff c_3 \in \ker(A-I)$
- $Ac_4 = c_3 + c_4$ | $(A-I)c_4 = c_3 \Rightarrow (A-I)^2 c_4 = 0 \iff c_4 \in \ker(A-I)^2$

$c_3 \neq 0 \Rightarrow c_4 \notin \ker(A-I)$



PROCEDIMENTO:

- c_1 elem non nullo quals di $\ker(A)$;
- c_4 elem di $\ker(A-I)^2$ ma non di $\ker(A-I)$;
- $c_3 = (A-I)c_4$ $[\Rightarrow c_3 \in \ker(A-I)]$;
- c_2 elem di $\ker(A-I)$ indip da c_3 (Oss: esiste ...!)

Concretam: $\ker(A) = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \right\rangle$, $\ker(A-I) = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\rangle$ (dimens = 2)

$\ker(A-I)^2 = \left\langle \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} \right\rangle$;

- $c_1 = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$
- $c_4 = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}$, $c_3 = (A-I)c_4 = \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$ $[\in \ker(A-I)]$, $c_2 = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 1 \end{pmatrix}$

Q.d.: $C = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

Es per casa: verif che $AC = C\,FCJ(A)$.

Es: $A = \begin{bmatrix} 2 & -1 & -2 & 3 \\ 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \in \mathbb{C}^{4\times 4}$;

- determ $FCJ(A)$;
- determ C t.c $AC = C\,FCJ(A)$ ...