Es: A t.c.
- $\sigma(A) = \{1, i\}$
- $m.a.(1) = 3$, $m.a.(i) = 2$
- $m.g.(1) = 2$, $m.g.(i) = 2$

$FCJ(A) = ?$

1) $p_A(x) = (1-x)^3 (i-x)^2 \Rightarrow A \in \mathbb{C}^{5\times5}$

2) # blocchi assoc a $\lambda = 1$: 2, $\delta_{11} + \delta_{12} = m.a.(1) = 3$

$\Rightarrow \delta_{11} = 1, \ \delta_{12} = 2$

3) # blocchi assoc a $\lambda = i$: 2, $\delta_{21} + \delta_{22} = 2 \Rightarrow \delta_{21} = \delta_{22} = 1$

Q.di: $FCJ(A) = \begin{pmatrix} 1 & & & & \\ & 1 & 1 & & \\ & & 1 & & \\ & & & i & \\ & & & & i \end{pmatrix}$

Es: A t.c.
- $\sigma(A) = \{1, i\}$
- $m.a.(1) = 3$, $m.a.(i) = 2$
- $m.g.(1) = 3$, $m.g.(i) = 2$

$FCJ(A) = ?$

Es: A t.c.
- $\sigma(A) = \{1, i\}$
- $m.a.(1) = 4$, $m.a.(i) = 2$
- $m.g.(1) = 2$, $m.g.(i) = 1$

$FCJ(A) = ?$

Sol:

1) $p_A(x) = (1-x)^4 (i-x)^2 \Rightarrow A \in \mathbb{C}^{6\times6}$

2) # blocchi assoc a $\lambda = 1$: 2, $\delta_{11} + \delta_{12} = 4 \Rightarrow$ $\delta_{11} = 1, \ \delta_{12} = 3$ / $\delta_{11} = 2, \ \delta_{12} = 2$

3) # blocchi assoc a $\lambda = i$: 1, $\delta_{21} = 2$

Quindi:

$FCJ(A) = \begin{pmatrix} 1 & & & & & \\ & 1 & 1 & & & \\ & & 1 & 1 & & \\ & & & 1 & & \\ & & & & i & 1 \\ & & & & & i \end{pmatrix}$ oppure $\begin{pmatrix} 1 & 1 & & & & \\ & 1 & & & & \\ & & 1 & 1 & & \\ & & & 1 & & \\ & & & & i & 1 \\ & & & & & i \end{pmatrix}$

> **Oss:** per decidere, basta guardare $\dim\ker\left(FCJ(A) - I\right)^2 \ldots$

Oss:

(1) A simile a $B \Rightarrow \dim\ker A = \dim\ker B$ (dim: ...)

(2) A simile a $FCJ(A) \Rightarrow A - \lambda I$ simile a $FCJ(A) - \lambda I$

$FCJ(A) = CAC^{-1} \Rightarrow FCJ(A) - \lambda I = CAC^{-1} - \lambda CC^{-1} = C(A - \lambda I)C^{-1}$

(3) A simile a $B \Rightarrow A^k$ simile a B^k (per $k \in \mathbb{Z}$)

$B = CAC^{-1} \Rightarrow B^2 = CAC^{-1}CAC^{-1} = CA^2C^{-1}$

Es: $A = \begin{pmatrix} 2 & -1 & -2 & 2 \\ 1 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{pmatrix} \in \mathbb{C}^{4\times4}$;

- $p_A(x) = (1-x)^3(-x)$

$* \ \sigma(A) = \{0, 1\}$, $m.a.(0) = 1$, $m.a.(1) = 3$

- $\dim\ker(A - 0I) = 1$, $m.g.(0) = 1$; $\delta_{11} = 1$
- $\dim\ker(A - I) = 2$, $m.g.(1) = 2$, $\delta_{21} + \delta_{22} = 3$; $\delta_{21} = 1, \ \delta_{22} = 2$

Q.di: $FCJ(A) = \mathrm{diag}\left(0, 1, \begin{pmatrix} 1 & 1 \\ & 1 \end{pmatrix}\right)$.

> **Es** (per casa): determ FCJ di $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 1 & -2 \\ 0 & 0 & 2 \end{pmatrix}$ (Sol: $FCJ(A) = \begin{pmatrix} 1 & & \\ & 2 & 1 \\ & & 2 \end{pmatrix}$)

Es: $A = \begin{pmatrix} 2 & 0 & 1 & 0 \\ 1 & 2 & 0 & 1 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 1 & 2 \end{pmatrix} \in \mathbb{C}^{4\times4}$;

- $p_A(x) = (2-x)^4$; $* \ \sigma(A) = \{2\}$, $m.a.(2) = 4$
- $\dim\ker(A - 2I) = 2$, $m.g.(2) = 2$; $\delta_{11} + \delta_{12} = 4$
- $\dim\ker(A - 2I)^2 = 3$, # blocchi di dimens ≥ 2 $= 3 - 2$; $\delta_{11} = 1, \ \delta_{12} = 3$

dunque: $FCJ(A) = \mathrm{diag}\left(2, \begin{pmatrix} 2 & 1 & \\ & 2 & 1 \\ & & 2 \end{pmatrix}\right)$.

Es. (per casa):

(1) decidere quali delle seg. matrici sono in FCJ:

$\begin{pmatrix} 1 & \\ & 7 \end{pmatrix}$; $\begin{pmatrix} 1 & & \\ & 3 & 2 \\ & 1 & 0 \end{pmatrix}$; $\begin{pmatrix} 3 & 1 & \\ & 6 & \\ & & 2 \end{pmatrix}$; $\begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & 1 \\ & & & 1 \end{pmatrix}$.

(2) $A, B \in \mathbb{C}^{4\times4}$ tali che

$FCJ(A) = \mathrm{diag}\left(2, \begin{pmatrix} i & 1 & \\ & i & 1 \\ & & i \end{pmatrix}\right)$; $FCJ(B) = \mathrm{diag}\left(\begin{pmatrix} i & 1 & \\ & i & 1 \\ & & i \end{pmatrix}, 2\right)$.

Decidere se A e B sono simili.