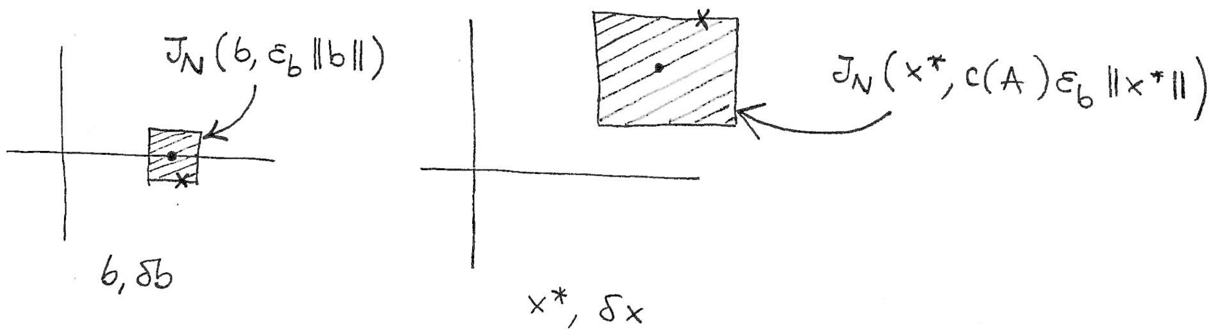
<u>Oss</u>:



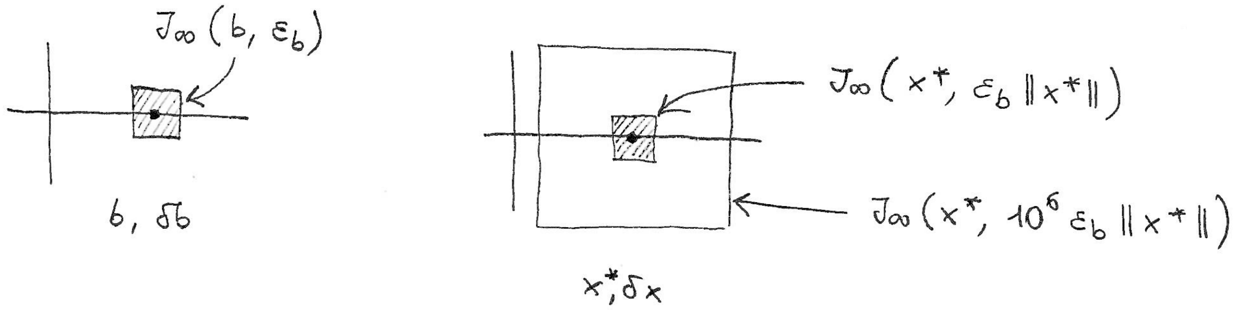
<u>Es</u>: $A = \begin{bmatrix} 10^{-3} & 0 \\ 0 & 10^{3} \end{bmatrix}$ • $\det A = 1$, $c_\infty(A) = 10^6$

• $x^* = \begin{bmatrix} 10^3 b_1 \\ 10^{-3} b_2 \end{bmatrix}$, $\hat{x} = x^* + \delta x = \begin{bmatrix} 10^3(b_1+\delta b_1) \\ 10^{-3}(b_2+\delta b_2) \end{bmatrix}$, $\delta x = \begin{bmatrix} 10^3 \delta b_1 \\ 10^{-3} \delta b_2 \end{bmatrix}$

(1) $b = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow x^* = \begin{pmatrix} 10^3 \\ 0 \end{pmatrix}$, $\|x^*\|_\infty = 10^3$

• $\varepsilon_b = \dfrac{\|\delta b\|_\infty}{\|b\|_\infty} = \|\delta b\|_\infty$

• $\|\delta x\|_\infty \leq 10^3 \|\delta b\|_\infty \Rightarrow \boxed{\varepsilon_d \leq \dfrac{10^3 \|\delta b\|_\infty}{10^3} = \|\delta b\|_\infty = \varepsilon_b}$

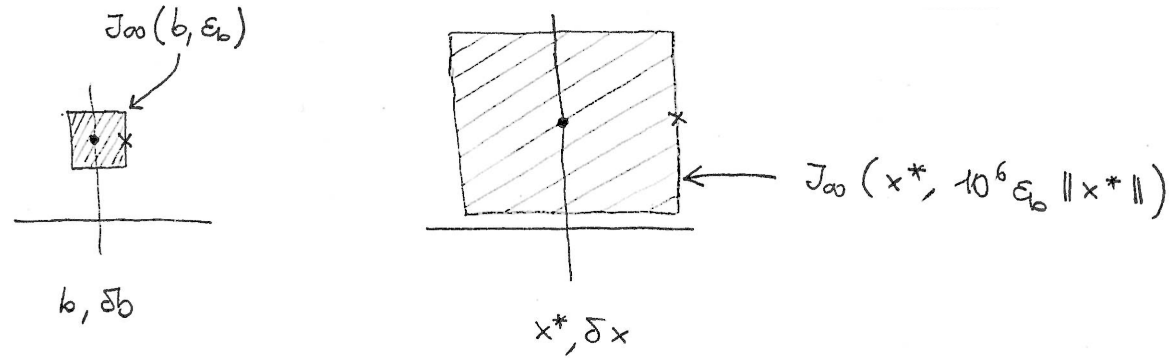


(2) $b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\delta b = \begin{pmatrix} \delta b_1 \\ 0 \end{pmatrix} \Rightarrow$ $x^* = \begin{pmatrix} 0 \\ 10^{-3} \end{pmatrix}$, $\|x^*\|_\infty = 10^{-3}$,

$\delta x = \begin{pmatrix} 10^3 \delta b_1 \\ 0 \end{pmatrix}$

• $\varepsilon_b = \|\delta b\|_\infty$

• $\|\delta x\|_\infty = 10^3 \|\delta b\|_\infty \Rightarrow \boxed{\varepsilon_d = \dfrac{10^3 \|\delta b\|_\infty}{10^{-3}} = 10^6 \|\delta b\|_\infty = 10^6 \varepsilon_b}$



• <u>CASO 2</u>: $\delta b = 0$, δA t.c. $A + \delta A$ invert

$$\hat{\varepsilon}_d = \frac{\|\delta x\|}{\|\hat{x}\|}$$

<u>TEO</u> (condiz. II): $A \in \mathbb{R}^{n \times n}$ invert;

• $\forall b \begin{cases} \in \mathbb{R}^n \\ \neq 0 \end{cases}$, $\forall \delta A \begin{cases} \in \mathbb{R}^{n\times n} \\ \text{t.c. } A+\delta A \text{ invert} \end{cases}$: $\hat{\varepsilon}_d \leq c(A)\, \varepsilon_A$

• $\exists b \begin{cases} \in \mathbb{R}^n \\ \neq 0 \end{cases}$, $\exists \delta A \begin{cases} \in \mathbb{R}^{n\times n} \\ \text{t.c. } A+\delta A \text{ invert} \end{cases}$: $\hat{\varepsilon}_d = c(A)\, \varepsilon_A$

<u>Oss</u>: $(\mathbb{R}^n, N)$, $A \in \mathbb{R}^{n\times n}$ invert: $c_N(A) \geq 1$

(<u>dim</u>: $I = A^{-1}A \Rightarrow 1 = \|I\|_N \leq \|A^{-1}\|_N \|A\|_N$)

Oss: (1) $\| A^{-1}\delta A \| < 1 \Rightarrow A + \delta A$ invert

dim: $A+\delta A = A\,(I + A^{-1}\delta A)$ invert $\Leftrightarrow I + A^{-1}\delta A$ invert $\Leftrightarrow \left[(I + A^{-1}\delta A)\, v = 0 \Leftrightarrow v = 0 \right]$

$\Leftrightarrow \left[A^{-1}\delta A\, v = -v \Leftrightarrow v = 0 \right]$

Per ass: SE $\exists v \neq 0$ t.c. $A^{-1}\delta A\, v = -v$

ALLORA $\|v\| = \|A^{-1}\delta A\, v\| \leq \|A^{-1}\delta A\| \, \|v\| < \|v\|$, assurdo.

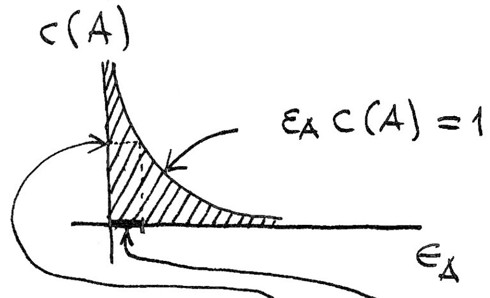
(2) $\epsilon_A\, c(A) < 1 \Rightarrow \|A^{-1}\delta A\| < 1$

e q.di: $A + \delta A$ invertibile

dim: $\epsilon_A\, c(A) < 1 \Leftrightarrow \|\delta A\|\, \|A^{-1}\| < 1$

e $\|A^{-1}\delta A\| \leq \|A^{-1}\|\, \|\delta A\|$ ...

(3)



data A, calcolo $c(A)$:

SE ϵ_A $\boxed{\text{suff piccolo}}$

ALLORA: $A + \delta A$ certamente invertibile.

Es: $x = \begin{bmatrix} 1 \\ 10^{-2} \end{bmatrix}$, δx: $\underbrace{\dfrac{\|\delta x\|_\infty}{\|x\|_\infty} \leq 10^{-3}}$

$\Downarrow$ $\|x\|_\infty = 1$

$\Downarrow$ $\|\delta x\|_\infty \leq 10^{-3}$

ovvero: $\max\{|\delta x_1|, |\delta x_2|\} \leq 10^{-3}$

Si ha:

- $\dfrac{|\delta x_1|}{|x_1|} = |\delta x_1| \leq 10^{-3}$ OK

$\left[|\delta x_k| \leq \|\delta x\|_\infty \right]$

- $\dfrac{|\delta x_2|}{|x_2|} = 10^2\, |\delta x_2| \leq 10^{-1}$!!

Es (per casa): $x = \begin{pmatrix} 100 \\ -1 \end{pmatrix}$, δx: $\dfrac{\|\delta x\|_2}{\|x\|_2} \leq 10^{-2}$

- determinare max di $\dfrac{|\delta x_1|}{|x_1|}$, $\dfrac{|\delta x_2|}{|x_2|}$.

Oss: dati A invert, $b \neq 0$, $\hat{x}$

- $r \equiv A\hat{x} - b$

  RESIDUO (associato ad $\hat{x}$)

- interpretazione di r ed $\hat{x}$:
  - $\hat{x}$ è LA soluzione del SISTEMA PERTURBATO $Ax = b + r$;
  - r è la perturbazione di b
- Teo condiz, I: $\epsilon_d \leq c(A)\, \epsilon_b$

  $\epsilon_d = \dfrac{\|\hat{x} - x^*\|}{\|x^*\|}$, $\quad \epsilon_b = \dfrac{\|r\|}{\|b\|}$