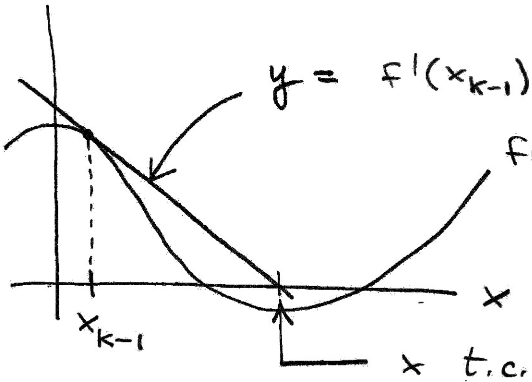
(3) Int geometrica (METODO delle TANGENTI):



$y = f'(x_{k-1})(x - x_{k-1}) + f(x_{k-1})$

(retta tangente al grafico di f in x_{k-1})

x t.c. $f'(x_{k-1})(x - x_{k-1}) + f(x_{k-1}) = 0$

<u>ovvero</u> : $x = x_{k-1} - \dfrac{f(x_{k-1})}{f'(x_{k-1})} = h(x_{k-1})$

(4) Scelta di x_0 per m di Newt:

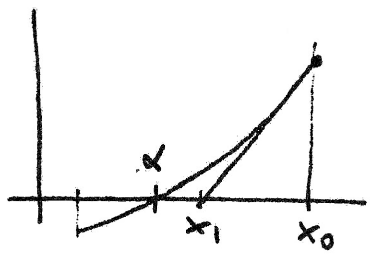
<u>SE</u> $[a,b]$, $f \in C^2[a,b]$, x_0 t.c.

1) $\exists\, \alpha \in [a,b]$ zero di f
2) $\forall x \in [a,b]$, $f'(x) \neq 0$ e $f''(x) \neq 0$ $[\Rightarrow \alpha$ unico zero$]$
3) $f(x_0)\, f''(x_0) > 0$

<u>ALLORA</u> : la success gen dal m di Newt a partire da x_0...

1) è CONVERGENTE ad α
2) è MONOTONA

<u>dim</u> : graficamente...



$\alpha < \ldots < x_1 < x_0$

<u>Es</u>: calcolo radici n-esima

$f(x) = x^n - b$, $b > 0$, $n \geq 2$

- CRITERI di ARRESTO (per m it def da h)

α pu, $|h'(\alpha)| < 1$, h regolare

$\boxed{1}$ dato $E > 0$: <u>se</u> $|x_k - x_{k-1}| < E$ <u>allora</u> STOP

(1.A) $[a,b]$ che verif ip (1) e (2) del Teo Conv,

$\forall x \in [a,b]$, $h'(x) < 0$

$\Rightarrow$ α tra x_k e x_{k-1}

e quindi: $|x_k - x_{k-1}| < E \Rightarrow |x_k - \alpha| < E$

(1.B) $\forall x \in [a,b]\ \exists \theta$ t.c. $h(x) = h(\alpha) + h'(\theta)(x - \alpha)$

$$\Rightarrow x_k - x_{k-1} = h(x_{k-1}) - x_{k-1}$$

$$= \alpha + h'(\theta_k)(x_{k-1} - \alpha) - x_{k-1}$$

$$= (x_{k-1} - \alpha)(h'(\theta_k) - 1)$$

$$\Rightarrow |x_k - x_{k-1}| = |x_{k-1} - \alpha|\,(1 + \varepsilon_k) \qquad \varepsilon_k \to -h'(\theta_k)$$

$\lvert x_k - x_{k-1}\rvert$ appross $\lvert x_{k-1} - \alpha\rvert$ con err rel ε_k e $\lvert\varepsilon_k\rvert \to \lvert h'(\alpha)\rvert < 1$.

<u>Es</u>: • $|h'(\alpha)| = \frac{1}{100}$; $|x_k - \alpha| = 10^{-4}$

$\Rightarrow |x_k - x_{k-1}| \in 10^{-4} \pm 10^{-6}$ (bene)

- $|h'(\alpha)| = \frac{9}{10}$; $|x_k - \alpha| = 10^{-4}$

$$\Rightarrow |x_k - x_{k-1}| \in 10^{-4}\left[\frac{1}{10}, \frac{19}{10}\right] \quad \text{(NON bene!)}$$

$\boxed{2}$ dato $E > 0$: <u>se</u> $\dfrac{|x_k - x_{k-1}|}{|1 - h'(x_k)|} < E$ <u>allora</u> STOP

$$\frac{x_k - x_{k-1}}{1 - h'(x_k)} = \frac{h'(\theta_k) - 1}{1 - h'(x_k)}(x_{k-1} - \alpha)$$

$$\Rightarrow \left|\frac{x_k - x_{k-1}}{1 - h'(x_k)}\right| = \underbrace{\left|\frac{1 - h'(\theta_k)}{1 - h'(x_k)}\right|}_{1 + \varepsilon_k,\ \varepsilon_k = \frac{h'(x_k) - h'(\theta_k)}{1 - h'(x_k)} \xrightarrow{k \to \infty} 0} |x_{k-1} - \alpha|$$

$$\boxed{\left|\frac{x_k - x_{k-1}}{1 - h'(x_k)}\right| \text{ approssima } |x_{k-1} - \alpha| \text{ con err rel} \to 0}$$

<u>Es</u>: $h(x) = x^2$.