- Criteri d'arresto (operando in $\mathbb{R}$)

1) h, $[a,b]$, x_0 che verif ip teo conv loc;

dato $\delta > 0$: $\boxed{\underline{\text{se}}\ |x_k - x_{k-1}| < \delta\ \underline{\text{allora}}\ \text{STOP}}$

infatti: • $|x_k - x_{k-1}| \begin{cases} \leq |x_k - \alpha| + |x_{k-1} - \alpha| \leq L^{k-1}(L+1)|x_0 - \alpha| \to 0 \\ = |h(x_{k-1}) - h(x_{k-2})| < |x_{k-1} - x_{k-2}| \end{cases}$

$\Rightarrow$ decrescenti

• $x_k - x_{k-1} = x_k - \alpha + \alpha - x_{k-1} = \underbrace{h(x_{k-1}) - h(\alpha)}_{= h'(\theta)(x_{k-1} - \alpha)} + \alpha - x_{k-1}$

$= (h'(\theta) - 1)(x_{k-1} - \alpha)$

$\Rightarrow \quad x_{k-1} - \alpha = \dfrac{x_k - x_{k-1}}{h'(\theta) - 1} \Rightarrow |x_{k-1} - \alpha| \leq \dfrac{|x_k - x_{k-1}|}{1 - L} < \dfrac{\delta}{1 - L}$

$\Rightarrow \quad \boxed{|x_k - \alpha| \leq L|x_{k-1} - \alpha| < \dfrac{L}{1-L}\delta}$

2) $f \in \mathcal{C}^1$, $f' \neq 0$ su $[a,b]$, $m = \min\{|f'(x)|,\ x \in [a,b]\}$

dato $\delta > 0$: $\underline{\text{se}}\ |f(x_k)| < \delta\ \underline{\text{allora}}$ STOP

infatti: • $x_k \to \alpha$ (dalle ip) $\Rightarrow f(x_k) \to 0$ (continuita' di f)

• $f(x_k) - f(\alpha) \begin{cases} = f(x_k) \\ = f'(\theta)(x_k - \alpha) \end{cases} \Rightarrow \quad x_k - \alpha = \dfrac{f(x_k)}{f'(\theta)}$

$\Rightarrow \quad |x_k - \alpha| \leq \dfrac{|f(x_k)|}{m} < \dfrac{\delta}{m}$

<u>Oss</u>: ① attenzione se $L \approx 1$; ② attenzione se $m \approx 0$.