- CONVERGENZA in $F(\beta, m)$

Teo: Siano

- h, $[a,b]$, x_0 che verificano le ip del Teo di convergenza;
- $\varphi : M \to M$ t.c. $\forall \xi \in [a,b] \cap M$, $|\varphi(\xi) - h(\xi)| \leq \delta$

$= F(\beta, m)$

- $\xi_k = \varphi(\xi_{k-1}) \in [a,b]$ per ogni k

p.u. di h in $[a,b]$

Allora: (1) $|\xi_k - \alpha| \leq \delta + L\,|\xi_{k-1} - \alpha|$

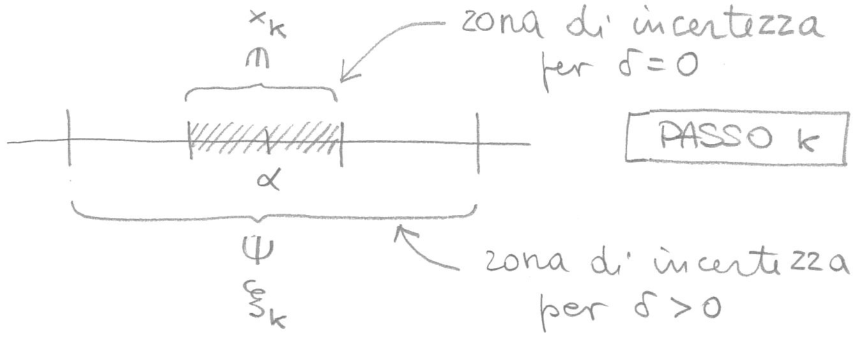
(2) $|\xi_k - \alpha| \leq \dfrac{\delta}{1-L} + L^k \left(|\xi_0 - \alpha| + \dfrac{\delta}{1-L} \right)$

dim: (1) $|\xi_k - \alpha| = |\varphi(\xi_{k-1}) - h(\alpha)| \leq |\varphi(\xi_{k-1}) - h(\xi_{k-1})| +$

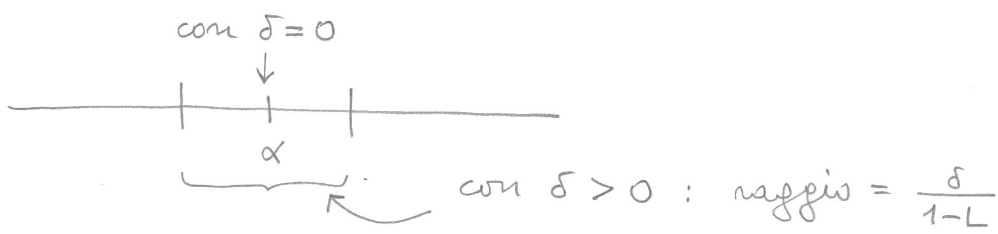
$+ |h(\xi_{k-1}) - h(\alpha)| \leq \delta + L\,|\xi_{k-1} - \alpha|$

(2) iterando il procedimento.

Ovvero:



$k \to \infty$



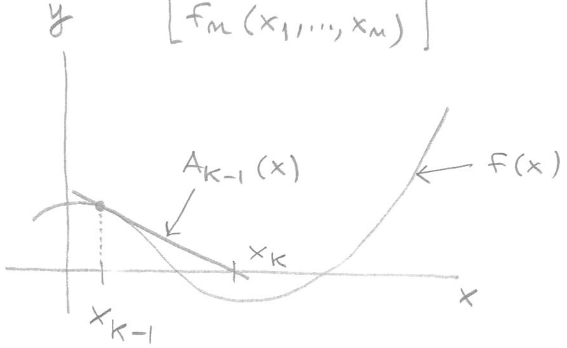
* METODO di NEWTON per $f: \mathbb{R}^n \to \mathbb{R}^n$ *

(1) Idea: ad ogni passo si affronta il problema "linearizzato"...

$$f(x) = \underbrace{f(x_{k-1}) + J_f(x_{k-1})(x - x_{k-1})}_{A_{k-1}(x)} + \dots$$

$$f(x) \to \begin{bmatrix} f_1(x_1, \dots, x_n) \\ \vdots \\ f_n(x_1, \dots, x_n) \end{bmatrix}$$

$n = 1$



$J_f(x) \in \mathbb{R}^{n \times n}$ MATRICE JACOBIANA di f in x...

$$\begin{bmatrix} \frac{\partial f_1}{\partial x_1}(x) & \dots & \frac{\partial f_1}{\partial x_n}(x) \\ \vdots & & \\ \frac{\partial f_n}{\partial x_1}(x) & \dots & \frac{\partial f_n}{\partial x_n}(x) \end{bmatrix}$$

SE $J_f(x_{k-1})$ non singolare

ALLORA: (I) si risolve il SISTEMA LINEARE $\boxed{J_f(x_{k-1})\, v = -f(x_{k-1})}$

(II) si pone $\boxed{x_k = x_{k-1} + v}$

Es: $f(x) = \begin{bmatrix} x_1^2 - x_2 \\ -x_1 + x_2^2 \end{bmatrix} : \mathbb{R}^2 \to \mathbb{R}^2$

- $J_f(x) = \begin{bmatrix} 2x_1 & -1 \\ -1 & 2x_2 \end{bmatrix} : \mathbb{R}^2 \to \mathbb{R}^{2 \times 2}$

- $x_0 = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$; sistema da risolvere: $J_f(x_0)\, v = -f(x_0)$

ovvero: $\begin{bmatrix} 2 & -1 \\ -1 & -2 \end{bmatrix} v = - \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

$$v = \begin{bmatrix} -4/5 \\ 2/5 \end{bmatrix} \Rightarrow x_1 = x_0 + v = \begin{bmatrix} 1/5 \\ -3/5 \end{bmatrix}$$

(2) È il metodo ad un punto def da

$$h(x) = x - J_f(x)^{-1} f(x) : \mathbb{R}^n \to \mathbb{R}^n$$

- TEO (convergenza LOCALE): Siano $h : \mathbb{R}^n \to \mathbb{R}^n$ suff regolare e $\alpha \in \mathbb{R}^n$ pu di h.

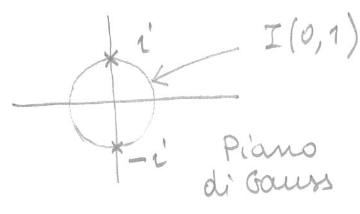
<u>SE</u> $\rho(J_h(\alpha)) < 1$

<u>ALLORA</u>: $\exists\, r > 0$ t.c. $\forall x_0 \in I(\alpha, r)$
si ha $\boxed{\lim_{k \to \infty} x_k = \alpha}$

<u>def</u> (SPETTRO, RAGGIO SPETTRALE)
Sia $M \in \mathbb{C}^{n \times n}$
(I) $\sigma(M) = \{ \lambda \in \mathbb{C} \mid \lambda \text{ autovalore di } M \}$
(II) $\rho(M) = \max\{ |\lambda|, \lambda \in \sigma(M) \}$

<u>Es</u>: $M = \begin{pmatrix} 2 & 1 \\ 0 & -1 \end{pmatrix}$; $\sigma(M) = \{2, -1\}$, $\rho(M) = 2$

$M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$; $\sigma(M) = \{i, -i\}$, $\rho(M) = 1 \longrightarrow$



$M = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$; $\sigma(M) = \{0\}$, $\rho(M) = 0$

- $h(x) = x - J_f(x)^{-1} f(x)$

$$\Rightarrow \quad J_h(x) = \cancel{I} - \divideontimes f(x) - \cancel{J_f(x)^{-1} J_f(x)}$$

$$\Rightarrow \quad \boxed{J_h(\alpha) = 0 \quad \text{se} \quad f(\alpha) = 0}$$

- TEO conv locale $\Rightarrow$ $\forall \alpha$ t.c. $f(\alpha) = 0$ e $J_f(\alpha)$ non singolare, $\exists\, r(\alpha)$ t.c.

$$\forall x_0 \in I(\alpha, r(\alpha)), \quad \lim_{k \to \infty} x_k = \alpha$$