

• PROPRIETA' delle NORME INDOTTE : \mathbb{R}^n, N

(I) $\forall A \in \mathbb{R}^{n \times n}, v \in \mathbb{R}^n : N(Av) \leq \|A\|_N N(v)$
 (dim: dalla def di norma indotta...)

(II) $\forall A, B \in \mathbb{R}^{n \times n} : \|AB\|_N \leq \|A\|_N \|B\|_N$
 (ragionevoli estensioni della precedenti)

(III) $\forall A \in \mathbb{R}^{n \times n}$:

• $\left\{ \frac{N(Av)}{N(v)}, v \neq 0 \right\} = \{ N(Av), N(v)=1 \}$

• $\|A\|_N = \max \{ N(Av), N(v)=1 \}$ (dim...)
 ($\Rightarrow \exists v_* \in \mathbb{R}^n$ t.c. $\begin{cases} N(v_*)=1 \\ N(Av_*) = \|A\|_N \end{cases}$)

• se A invertibile:

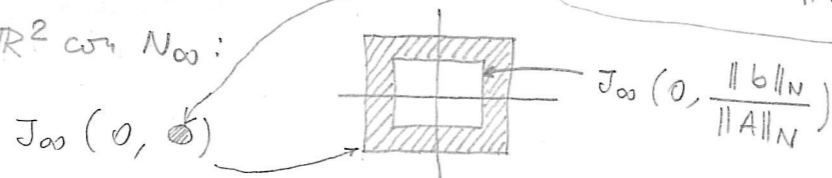
$\|A^{-1}\|_N = \left(\min \{ N(Av), N(v)=1 \} \right)^{-1}$

Es: • $A \in \mathbb{R}^{n \times n}$ invert; $A^{-1}A = I \Rightarrow \begin{cases} \|A^{-1}\|_N \|A\|_N \geq 1 \\ \|A^{-1}\|_N \geq \|A\|_N^{-1} \end{cases}$

• $A \in \mathbb{R}^{n \times n}$ invert, $b \in \mathbb{R}^n, x_* \in \mathbb{R}^n$ t.c. $Ax_* = b$

$\Rightarrow \frac{\|b\|_N}{\|A\|_N} \leq \|x_*\| \leq \|A^{-1}\|_N \|b\|_N = \|A\|_N \|A^{-1}\|_N \frac{\|b\|_N}{\|A\|_N}$

(Ad. es, \mathbb{R}^2 con N_∞ :



• CONDIZIONAMENTO •

dati: $A \in \mathbb{R}^{n \times n}$ invert, $b \in \mathbb{R}^n$

soluzione: $x_* \in \mathbb{R}^n : Ax_* = b$

dati PERTURBATI: $A' \in \mathbb{R}^{n \times n}$ invert (ragionevole...)
 $b' \in \mathbb{R}^n$

$\delta_A = A' - A, \delta_b = b' - b$: PERTURBAZ

soluzione: $x'_* \in \mathbb{R}^n : A'x'_* = b'$

Allora: $x'_* - x_* = (A + \delta_A)^{-1} (b + \delta_b) - A^{-1}b = F(A, b; \delta_A, \delta_b)$

Pb: Studia F

• CASO 1: $\delta_A = 0, \delta_b \neq 0$

$x'_* = A^{-1}(b + \delta_b), x'_* - x_* = A^{-1}\delta_b$
 $\Rightarrow \|x'_* - x_*\| = \|A^{-1}\delta_b\|$; $\left. \begin{matrix} \text{ERRORE TRASM} \\ \text{dai DATI...} \end{matrix} \right\} \varepsilon_d = \frac{\|x'_* - x_*\|}{\|x_*\|} = \frac{\|A^{-1}\delta_b\|}{\|A^{-1}b\|} \dots$

$\dots \leq \|A^{-1}\| \frac{\|\delta_b\|}{\|b\|} \|b\| \leq \|A^{-1}\| \|A\| \varepsilon_b$
 $= \varepsilon_b$ (ERRORE rel SUL DATO)

$A(A^{-1}b) = b \Rightarrow \|b\| \leq \|A\| \|A^{-1}b\|$
 $\Rightarrow \frac{\|b\|}{\|A^{-1}b\|} \leq \|A\|$

def (numero di condiz di A): $A \in \mathbb{R}^{n \times n}$ invert

$$c_N(A) = \|A\|_N \|A^{-1}\|_N \text{ numero di condiz di } A \text{ (in norma } N)$$

Teo (condiz, I): $A \in \mathbb{R}^{n \times n}$, invert

$$\bullet \forall b \begin{cases} \in \mathbb{R}^n \\ \neq 0 \end{cases}, \forall \delta_b \in \mathbb{R}^n: \varepsilon_d \leq c(A) \varepsilon_b$$

$$\bullet \exists b \begin{cases} \in \mathbb{R}^n \\ \neq 0 \end{cases}, \exists \delta_b \in \mathbb{R}^n: \varepsilon_d = c(A) \varepsilon_b$$