

Es (f di condizionamenti per f elementari):

$$f: \mathbb{R} \rightarrow \mathbb{R} \quad f \text{ elem}, \quad x \in \mathbb{R}, \quad f(x) \neq 0$$

$$C(x; \varepsilon) = \frac{f((1+\varepsilon)x) - f(x)}{f(x)} = f'(\theta) \frac{x}{f(x)} \varepsilon$$

(se f suff regular, θ tra x e $(1+\varepsilon)x$)

$$\bullet \lim_{\varepsilon \rightarrow 0} f'(\theta) = f'(x) \Rightarrow C(x; \varepsilon) \approx f'(x) \frac{x}{f(x)} \varepsilon$$

A) $f(x) = e^x$, $C(x; \varepsilon) \approx x\varepsilon$

• calcolo non ben condizionato per $|x|$ grande

B) $f(x) = \sin x$, $C(x; \varepsilon) \approx \frac{x}{\tan x} \varepsilon$

• calcolo non ben condizionato per $|x|$ grande
e $x \approx$ zero non nullo di $\sin x$

C) $f(x) = \sqrt{x}$, $C(x; \varepsilon) \approx \frac{1}{2} \varepsilon$

• calcolo sempre ben condizionato.

Oss: $n=1$, per semplicità, $x \in \mathbb{R}$, $\varepsilon \in \mathbb{R}$ ($f(x) \neq 0$)

$$C(x; \varepsilon) = \frac{f((1+\varepsilon)x) - f(x)}{f(x)} \Rightarrow f((1+\varepsilon)x) = (1 + C(x; \varepsilon)) f(x)$$

• calcolo ben condizionato significa: $\varepsilon \approx 0 \Rightarrow C(x; \varepsilon) \approx 0$

ovvero: $x' \approx x \Rightarrow f(x') \approx f(x)$, in senso relativo

Es (per caso): studiare il condiz del calcolo di

$$x^2, \quad \frac{1}{x}, \quad \log x$$

II) studio di ε_a : STABILITÀ dell'algo

Oss (caso elementari):

$$f: \mathbb{R}^n \rightarrow \mathbb{R}, \quad \varphi: F(\beta, m)^n \rightarrow F(\beta, m)$$

$$\text{t.c. } \forall \xi_1, \dots, \xi_m: \varphi(\xi_1, \dots, \xi_m) = \text{rd}(f(\xi_1, \dots, \xi_m))$$

Allora: $|\varepsilon_a| = \left| \frac{\varphi(\xi_1, \dots, \xi_m) - f(\xi_1, \dots, \xi_m)}{f(\xi_1, \dots, \xi_m)} \right| \leq u$

Es: pseudo-op aritmetiche, f prefef corrisp alle f elementari.

Es (casi non elem)

(A) $f(x) = \sin x + \cos x$, $\varphi(\xi) = \text{SEN}(\xi) \oplus \text{COS}(\xi)$

• $\text{SEN}(\xi) = \text{rd}(\sin \xi) = (1 + \varepsilon_1) \sin \xi$, $|\varepsilon_1| \leq u$

• $\text{COS}(\xi) = \text{rd}(\cos \xi) = (1 + \varepsilon_2) \cos \xi$, $|\varepsilon_2| \leq u$

• $\eta_1 \oplus \eta_2 = \text{rd}(\eta_1 + \eta_2) = (1 + \varepsilon_3)(\eta_1 + \eta_2)$, $|\varepsilon_3| \leq u$

$$\begin{aligned} \Rightarrow \varphi(\xi) &= (1 + \varepsilon_3) \left((1 + \varepsilon_1) \sin \xi + (1 + \varepsilon_2) \cos \xi \right) \\ &= (1 + \varepsilon_3) (1 + \varepsilon_1) \sin \xi + (1 + \varepsilon_3) (1 + \varepsilon_2) \cos \xi \\ &= (1 + \theta_1) \sin \xi + (1 + \theta_2) \cos \xi \end{aligned}$$

$$\Rightarrow \varepsilon_a = \frac{\varphi(\xi) - f(\xi)}{f(\xi)} = C(\sin \xi, \cos \xi; \theta_1, \theta_2)$$

↳ f di condiz di $x_1 + x_2$.

$$\text{def } \underbrace{S(\xi; \varepsilon_1, \varepsilon_2, \varepsilon_3)}_{f \text{ di stabilit\`a di } \varphi} = C(\sin \xi, \cos \xi; \theta_1, \theta_2)$$

$$(B) \quad f(x) = x(x+1) \quad , \quad \varphi(\xi) = \xi \otimes^2 (\xi \oplus 1)$$

$$[\text{OM: } 1 \in F(\mathcal{B}, m)]$$

$$\varphi(\xi) = (1+\varepsilon_2) \xi (1+\varepsilon_1)(\xi+1) \quad \text{con } |\varepsilon_1|, |\varepsilon_2| \leq u$$

$$= (1+\varepsilon_2)(1+\varepsilon_1) \xi(\xi+1)$$

$$= (1+\theta) f(\xi)$$

$$\Rightarrow \varepsilon_a = \frac{\varphi(\xi) - f(\xi)}{f(\xi)} = \theta \left[\stackrel{\text{def}}{=} S(\xi; \varepsilon_1, \varepsilon_2) \right]$$

Obs: • casi elementari $|\varepsilon_a| \approx u$: φ STABILE

- (A) : φ STABILE se $|C(\sin \xi, \cos \xi; \theta_1, \theta_2)| \approx u$
ovvero se il calcolo di $\sin \xi + \cos \xi$ \(\grave{e}\) ben condizionato
- (B) : $|\varepsilon_a| \approx u$: sempre stabile.

E (per caso):

$$1) \quad x = \frac{2}{5}; \quad \text{esp e frez in base tre; } \text{rd}(x) \text{ in } F(3,2)$$

$$[\text{Ris}p: \quad \frac{2}{5} = 3^0 \cdot \frac{2}{5}; \quad \text{rd}(x) = 4/9]$$

$$2) \quad M = F(2,4)$$

$$\cdot \xi \in M \text{ con } \text{esp} \geq 4 \Rightarrow \xi \in \mathbb{Z}$$

$$\cdot \text{determ } \max \{ \xi \in M \mid \xi > 0 \text{ e } \xi \notin \mathbb{Z} \}$$

$$[\text{Ris}p: \quad \max = 2^3 0,111]$$

$$3) \quad M = F(10,3); \quad \text{determ } \{ \xi \in M \mid \xi \oplus 1 > 1 \}$$

$$[\text{Ris}p: \quad \xi \geq 10^{-2} 0,501]$$