CN # 56 / 31 maggio 2013 / A 11

Es:

$$\dot{x} = Ax \xrightarrow{\text{EULERO}} x_k = x_{k-1} + \underbrace{h_{k-1}}_{>0} A x_{k-1}$$

$$= \underbrace{(I + h_{k-1} A)}_{B} x_{k-1}$$

A inv

[$\Rightarrow$ $x=0$ unica conf ep]

Pti uniti:

$$\cancel{x} = \cancel{x} + h_{k-1} A x$$

$$0 = h_{k-1} A x$$

$$\Rightarrow \boxed{x=0 \text{ e' l' UNICO}}$$

- Ip: A diagonalizz (semplifica, ma non e' necess per il ris...):

$$I + h_{k-1} A = I + h_{k-1} S \Lambda S^{-1} =$$

$$= S \underbrace{(I + h_{k-1} \Lambda)}_{\mathrm{diag}(1+h_{k-1}\lambda_i)} S^{-1}$$

e' diagonalizz, e la matrice che realizza la diag e' S

- $h > 0$ dato:

$\lambda \in \mathbb{C}$ autoval di A

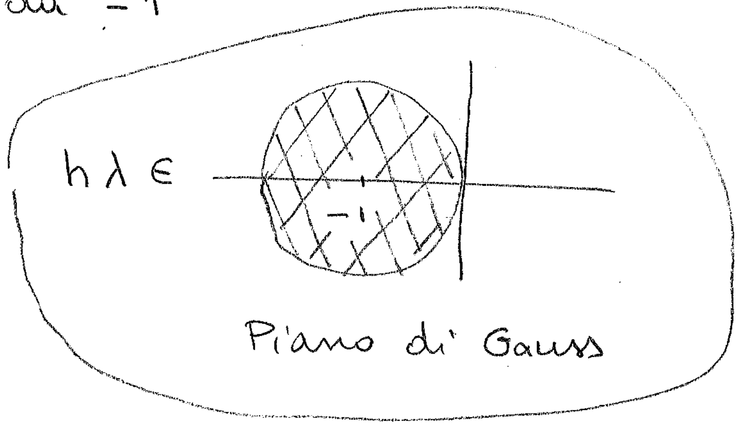
$\iff$ $1+h\lambda$ autoval di $I + hA$

[inoltre: m.a.(λ) = m.a.$(1+h\lambda)$ & mg(λ) = mg$(1+h\lambda)$]

- $h > 0$, $\lambda \in \mathbb{C}$ dati

$$|1 + h\lambda| = \underbrace{|h\lambda - (-1)|}_{\text{distanza di } h\lambda \text{ da } -1} \leq 1$$

$\Longleftrightarrow$



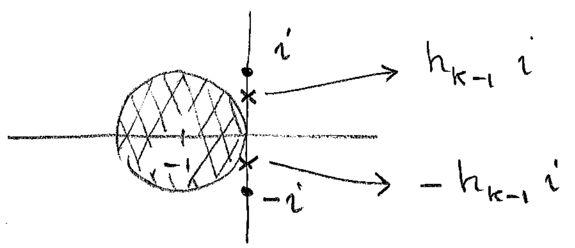
Oscill $\quad \dot{x} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x$

- autovalori : $\lambda_1 = i$, $\lambda_2 = -i$
- $re(\lambda_k) = 0$ E $ma(\lambda_k) = mg(\lambda_k) = 1$

  $\Rightarrow$ $x = 0$ è conf eq STABILE
  (ma NON as stabile...)

Eulero : $\quad x_k = x_{k-1} + h_{k-1} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x_{k-1}$

- autovalori : $\quad \lambda'_1 = 1 + h_{k-1} i$; $\lambda'_2 = 1 - h_{k-1} i$



$\boxed{\text{INSTABILE} \quad \forall\, h_{k-1} > 0}$

Es: $\dot{x} = Ax \xrightarrow{\text{EULERO IMPLICITO}} x_k = x_{k-1} + h_{k-1} A x_k$

$$\Rightarrow x_k = \underbrace{(I - h_{k-1}A)^{-1}}_{B} x_{k-1}$$

A inv $\Rightarrow$ ∃! ep sia per ep diff che in discreto

Ip: A diagonalizz ...

- $I - h_{k-1}A = S(I - h_{k-1}\Lambda)S^{-1}$

∃ ⟺ $\forall i$, $1 - h_{k-1}\lambda_i \neq 0$

$$\Rightarrow (I - h_{k-1}A)^{-1} = S\,\underbrace{(I - h_{k-1}\Lambda)^{-1}}\,S$$

$$= \mathrm{diag}\left(\frac{1}{1-h_{k-1}\lambda_1}, \dots, \frac{1}{1-h_{k-1}\lambda_n}\right)$$

$$\boxed{\lambda_i \longrightarrow \frac{1}{1-h_{k-1}\lambda_i}}$$

autov di A

autov di B , $\neq 0 \ \forall h_{k-1}$

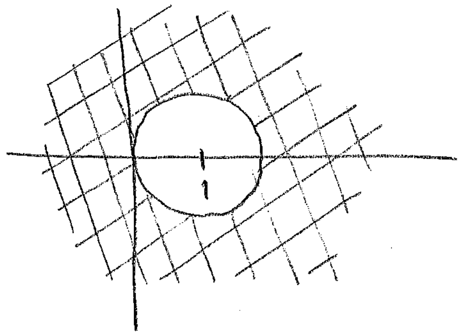
- sistema discreto:

  (1) ∃ ⟺ $\forall k, \forall i$: $h_{k-1}\lambda_i \neq 1$;

  (2) B , se esiste , è invertibile

- $h > 0$, $\lambda \in \mathbb{C}$ dati

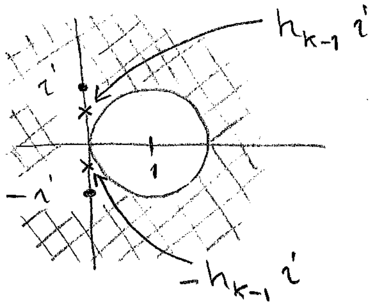
$$\left|\frac{1}{1-h\lambda}\right| \leq 1 \iff |1 - h\lambda| = \underbrace{|h\lambda - 1|}_{\text{distanza di } h\lambda \text{ da } 1} \geq 1$$

$\iff$ $h\lambda \in$ 

Piano di Gauss

OSCILL Eulero implicito :

- autovalori : $\lambda_1'' = \dfrac{1}{1 - h_{k-1}\, i}$, $\lambda_2'' = \dfrac{1}{1 + h_{k-1}\, i}$



$$\boxed{\begin{array}{c}\text{AS. STABILE} \\ \forall\, h_{k-1} > 0\end{array}}$$