Oss (ricostr mediante f cont lin a tratti)

$[a,b]$, $a=t_0,\dots,t_k=b$, S f cont lin a tratti su $I_1=[t_0,t_1],\dots$

• $r: \mathbb{R}^{k+1} \to \mathcal{C}([a,b],\mathbb{R})$ t.c. $r: \begin{bmatrix} y_0 \\ \vdots \\ y_k \end{bmatrix} \to$ l'elem di S che int $(t_0,y_0),\dots,(t_k,y_k)$

(1) r è f di ricostr rel a c (dim ...)

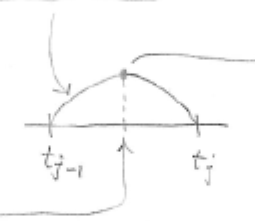
(2) $f\in\mathcal{C}^2([a,b],\mathbb{R})$, $M_2=\max\{\,|f^{(2)}(t)|,\ t\in[a,b]\}$

$\forall\, t\in I_j$, $|f(t)-r(c(f))(t)| = |f(t)-p_j(t)| \leq$ usando teo err ricostr int polinomiale

p_j: l'elem di $\mathbb{P}_1(\mathbb{R})$ che int $(t_{j-1}, f(t_{j-1})), (t_j, f(t_j))$

$$\leq \frac{M_2}{2}\underbrace{|t-t_{j-1}|\,|t-t_j|} \leq \frac{M_2}{2}\left(\frac{t_j-t_{j-1}}{2}\right)^2$$

$\frac{t_{j-1}+t_j}{2}$



$\Rightarrow$ posto $h(k)=\max\{t_1-t_0,\ t_2-t_1,\dots,t_k-t_{k-1}\}$

si ha: $\boxed{e(f)\leq \frac{M_2}{8}h(k)^2}$

> **Oss**: $\forall f\in\mathcal{C}^2([a,b],\mathbb{R})$: se strategia di scelta degli ist di camp è t.c. $\lim_{k\to\infty} h(k)=0$ allora $\lim_{k\to\infty} e(f)=0$

Es: • $t_j = a+\frac{b-a}{k}j$, $j=0,\dots,k$

$h(k)=\frac{b-a}{k} \Rightarrow \lim_{k\to\infty} h(k)=0$

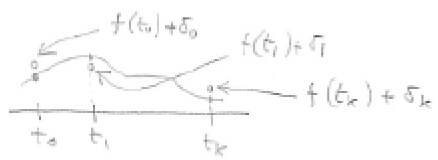
• $[a,b]=[0,1]$, $t_j=\frac{j}{j+1}$, $j=0,\dots,k-1$, $t_k=1$

$h(k)=1/2 \Rightarrow \lim_{k\to\infty} h(k)\neq 0$

Oss (condiz del pb della ricostruzione):

$[a,b]$; $t_0,\dots,t_k$; c f di camp; r f di ricostr

$\delta_0,\dots,\delta_k\in\mathbb{R}$; $f:[a,b]\to\mathbb{R}$



• $\hat{r}(t) = r(c(f)+\delta)$

• $\underbrace{|\hat{r}(t)-r(c(f))|}_{\text{errore assoluto, all'ist } t\text{, commesso ricostruendo dati "errati"}} = |r(\delta)|$

(A) ricostr con int polin: base di Lagrange di $\mathbb{P}_k(\mathbb{R})$

$$|r(\delta)| = |\delta_0\ell_0(t)+\dots+\delta_k\ell_k(t)|$$

$$\leq |\delta_0|\,|\ell_0(t)|+\dots+|\delta_k|\,|\ell_k(t)|$$

$$\leq \max\{|\delta_0|,\dots,|\delta_k|\}\underbrace{(|\ell_0(t)|+\dots+|\ell_k(t)|)}$$

$\max\{|\ell_0(t)|+\dots+|\ell_k(t)|,\ t\in[a,b]\}$ $\geq C\log k$

> **Oss**: $\exists\, t\in[a,b]$, $\delta_0,\dots,\delta_k\in\mathbb{R}$ t.c.
> $|r(\delta)| = \max|\delta_j|\,\max\{|\ell_0|+\dots+|\ell_k|\}$