Es: $A = \begin{bmatrix} 10^{-3} & 0 \\ 0 & 10^{3} \end{bmatrix}$; $\det A = 1$, $c_\infty(A) = 10^6$ $(\gg 1\,!)$

- $x^* = \begin{bmatrix} 10^3 b_1 \\ 10^{-3} b_2 \end{bmatrix}$, $\hat{x} = x^* + \delta x = \begin{bmatrix} 10^3 (b_1 + \delta b_1) \\ 10^{-3}(b_2 + \delta b_2) \end{bmatrix}$, $\delta x = \begin{bmatrix} 10^3 \delta b_1 \\ 10^{-3} \delta b_2 \end{bmatrix}$

(1) $b = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow x^* = \begin{pmatrix} 10^3 \\ 0 \end{pmatrix}$, $\|x^*\|_\infty = 10^3$

- $\epsilon_b = \dfrac{\|\delta b\|_\infty}{\|b\|_\infty} = \|\delta b\|_\infty$, $\|\delta x\|_\infty \le 10^3 \|\delta b\|_\infty$

$$\Rightarrow \epsilon_d = \frac{\|\delta x\|_\infty}{\|x^*\|_\infty} \le \frac{10^3 \|\delta b\|_\infty}{10^3} = \|\delta b\|_\infty = \epsilon_b$$

- Teo condiz, I $\Rightarrow$ $\epsilon_d \le 10^6 \epsilon_b$: molto pessimista !

(2) $b = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, $\delta b = \begin{pmatrix} \delta b_1 \\ 0 \end{pmatrix} \Rightarrow x^* = \begin{pmatrix} 0 \\ 10^{-3} \end{pmatrix}$, $\|x^*\|_\infty = 10^{-3}$,

$\delta x = \begin{pmatrix} 10^3 \delta b_1 \\ 0 \end{pmatrix}$

- $\epsilon_b = \|\delta b\|_\infty$, $\|\delta x\|_\infty = 10^3 \|\delta b\|_\infty$

$$\Rightarrow \epsilon_d = \frac{\|\delta x\|_\infty}{\|x^*\|_\infty} = 10^3 \frac{\|\delta b\|_\infty}{10^{-3}} = 10^6 \|\delta b\|_\infty = 10^6 \epsilon_b$$

- Teo condiz, I non pessimista !

- CASO 2 : $\delta b = 0$, δA t.c. $A + \delta A$ invert

$$\epsilon_A = \frac{\|\delta A\|}{\|A\|} , \quad \hat{\epsilon}_d = \frac{\|\delta x\|}{\|\hat{x}\|}$$

TEO (condiz, II) : $A \in \mathbb{R}^{n \times n}$, invert ;

- $\forall b \in \mathbb{R}^n,\ b \neq 0$, $\forall \delta A \in \mathbb{R}^{n\times n}$ t.c. $A+\delta A$ invert : $\hat{\epsilon}_d \le c(A)\, \epsilon_A$
- $\exists b \in \mathbb{R}^n,\ b \neq 0$, $\exists \delta A \in \mathbb{R}^{n\times n}$ t.c. $A+\delta A$ invert : $\hat{\epsilon}_d = c(A)\, \epsilon_A$

Oss : $\mathbb{R}^n$, N ; $A \in \mathbb{R}^{n\times n}$ invert : $c(A) \ge 1$

(dim : $I = A^{-1} A \Rightarrow 1 = \|I\| \le \|A^{-1}\|\, \|A\|$)

Es : Utilizz, in Octave, il comando

$$[A, B, C] = \text{lu}\,(M)$$

si ottiene :

$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & -1 & 0 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & -2 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

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1) determ M

2) ris il sist $Mx = \begin{pmatrix} 1 \\ 0 \\ 1 \\ 1 \end{pmatrix}$