- Criteri d'arresto (operando in $\mathbb{R}$)

1) h, $[a,b]$, x_0 che verif ip teo conv loc;

dato $\delta > 0$: $\boxed{\underline{\text{se}} \quad |x_k - x_{k-1}| < \delta \quad \underline{\text{allora}} \quad \text{STOP}}$

infatti: • $$|x_k - x_{k-1}| \begin{cases} \leq |x_k - \alpha| + |x_{k-1} - \alpha| \leq L^{k-1}(L+1)|x_0 - \alpha| \rightarrow 0 \\ = |h(x_{k-1}) - h(x_{k-2})| < |x_{k-1} - x_{k-2}| \end{cases}$$

$\Rightarrow$ <u>decrescenti</u>

• $$x_k - x_{k-1} = x_k - \alpha + \alpha - x_{k-1} = \underbrace{h(x_{k-1}) - h(\alpha)}_{= h'(\theta)(x_{k-1} - \alpha)} + \alpha - x_{k-1}$$

$$= \left(h'(\theta) - 1\right)(x_{k-1} - \alpha)$$

$$\Rightarrow \quad x_{k-1} - \alpha = \frac{x_k - x_{k-1}}{h'(\theta) - 1} \Rightarrow \boxed{|x_{k-1} - \alpha| \leq \frac{|x_k - x_{k-1}|}{1 - L} < \frac{\delta}{1-L}}$$

2) $f \in \mathcal{C}^1$, $f' \neq 0$ su $[a,b]$, $m = \min\left\{|f'(x)|,\ x \in [a,b]\right\}$

dato $\delta > 0$: <u>se</u> $|f(x_k)| < \delta$ <u>allora</u> STOP

infatti: • $$f(x_k) - f(\alpha) \begin{cases} = f(x_k) \\ = f'(\theta)(x_k - \alpha) \end{cases} \Rightarrow x_k - \alpha = \frac{f(x_k)}{f'(\theta)}$$

$$\Rightarrow |x_k - \alpha| \leq \frac{|f(x_k)|}{m} < \frac{\delta}{m}$$

• $x_k \to \alpha$ (dalle ip) $\Rightarrow f(x_k) \rightarrow 0$ (continuità di f)

<u>Oss</u>: ① attenzione se $L \approx 1$; ② attenzione se $m \approx 0$.