Es (f di condizionam per f elementari):

$f : \mathbb{R} \to \mathbb{R}$ f elem, $x \in \mathbb{R}$, $f(x) \neq 0$

$$C(x;\varepsilon) = \frac{f((1+\varepsilon)x) - f(x)}{f(x)} = f'(\theta)\,\frac{x}{f(x)}\,\varepsilon$$

(se f suff regolare, θ tra x e $(1+\varepsilon)x$)

- $\lim_{\varepsilon \to 0} f'(\theta) = f'(x) \Rightarrow \boxed{C(x;\varepsilon) \approx f'(x)\,\frac{x}{f(x)}\,\varepsilon}$

A) $f(x) = e^x$, $C(x;\varepsilon) \approx x\varepsilon$

- calcolo non ben condiz per $|x|$ grande

B) $f(x) = \operatorname{sen} x$, $C(x;\varepsilon) \approx \dfrac{x}{\operatorname{tg} x}\,\varepsilon$

- calcolo non ben condiz per $|x|$ grande
  e $x \approx$ zero non nullo di sen x

c) $f(x) = \sqrt{x}$, $C(x;\varepsilon) \approx \frac{1}{2}\varepsilon$

- calcolo sempre ben condizionato.

> Oss: $n=1$, per semplicita', $x \in \mathbb{R}$, $\varepsilon \in \mathbb{R}$ ($f(x) \neq 0$)
>
> $$C(x;\varepsilon) = \frac{f((1+\varepsilon)x) - f(x)}{f(x)} \Rightarrow f((1+\varepsilon)x) = (1 + C(x;\varepsilon))\,f(x)$$
>
> - calcolo ben condiz significa: $\varepsilon \approx 0 \Rightarrow C(x;\varepsilon) \approx 0$
>
> ovvero: $x' \approx x \Rightarrow f(x') \approx f(x)$, in senso relativo

Es (per casa): studiare il condiz del calcolo di

x^2, $\frac{1}{x}$, $\log x$

II) Studio di ε_a: STABILITA' dell'algo

Oss (caso elementare):

$f : \mathbb{R}^n \to \mathbb{R}$, $\varphi : F(\beta,m)^n \to F(\beta,m)$

t.c. $\forall\, \xi_1,\dots,\xi_n$: $\varphi(\xi_1,\dots,\xi_n) = rd(f(\xi_1,\dots,\xi_n))$

Allora: $$|\varepsilon_a| = \left| \frac{\varphi(\xi_1,\dots,\xi_n) - f(\xi_1,\dots,\xi_n)}{f(\xi_1,\dots,\xi_n)} \right| \leq u$$

> Es: pseudo-op aritmetiche, f predef corrisp alle f elementari.

Es (casi non elem)

(A) $f(x) = \operatorname{sen} x + \cos x$, $\varphi(\xi) = SEN(\xi) \oplus COS(\xi)$

- $SEN(\xi) = rd(\operatorname{sen}\xi) = (1+\varepsilon_1)\operatorname{sen}\xi$, $|\varepsilon_1| \leq u$
- $COS(\xi) = rd(\cos\xi) = (1+\varepsilon_2)\cos\xi$, $|\varepsilon_2| \leq u$
- $\eta_1 \oplus \eta_2 = rd(\eta_1 + \eta_2) = (1+\varepsilon_3)(\eta_1+\eta_2)$, $|\varepsilon_3| \leq u$

$$\Rightarrow \varphi(\xi) = (1+\varepsilon_3)\big((1+\varepsilon_1)\operatorname{sen}\xi + (1+\varepsilon_2)\cos\xi\big)$$
$$= (1+\varepsilon_3)(1+\varepsilon_1)\operatorname{sen}\xi + (1+\varepsilon_3)(1+\varepsilon_2)\cos\xi$$
$$= (1+\theta_1)\operatorname{sen}\xi + (1+\theta_2)\cos\xi$$

$$\Rightarrow \varepsilon_a = \frac{\varphi(\xi) - f(\xi)}{f(\xi)} = C(\operatorname{sen}\xi, \cos\xi; \theta_1, \theta_2)$$

$\hookrightarrow$ f di condiz di $x_1 + x_2$.

$\underline{\text{def}}$ $\underbrace{S(\xi;\ \varepsilon_1, \varepsilon_2, \varepsilon_3)}_{f \text{ di stabilità di } \varphi} = C(\text{sen}\,\xi,\ \cos\xi;\ \theta_1, \theta_2)$

(B) $f(x) = x(x+1)$, $\varphi(\xi) = \xi \overset{2}{\otimes} (\xi \overset{1}{\oplus} 1)$

[$\underline{\text{Oss}}$: $1 \in F(\beta, m)$]

$$\varphi(\xi) = (1+\varepsilon_2)\ \xi\ (1+\varepsilon_1)(\xi+1) \quad \text{con } |\varepsilon_1|, |\varepsilon_2| \leq u$$
$$= (1+\varepsilon_2)(1+\varepsilon_1)\ \xi(\xi+1)$$
$$= (1+\theta)\ f(\xi)$$
$$\Rightarrow \varepsilon_a = \frac{\varphi(\xi) - f(\xi)}{f(\xi)} = \theta \quad \left[\overset{\text{def}}{=} S(\xi;\ \varepsilon_1, \varepsilon_2) \right]$$

$\underline{\text{Oss}}$: • casi elementari $|\varepsilon_a| \approx u$: φ STABILE

• (A) : φ STABILE se $|C(\text{sen}\,\xi, \cos\xi;\ \theta_1, \theta_2)| \approx u$
ovvero se il calcolo di $\text{sen}\,\xi + \cos\xi$ è ben condiz

• (B) : $|\varepsilon_a| \approx u$: sempre stabile.

$\underline{\text{Es}}$ (per casa):

1) $x = \frac{2}{5}$; esp e fraz in base tre ; $rd(x)$ in $F(3,2)$

[$\underline{\text{Risp}}$: $\frac{2}{5} = 3^0 \cdot \frac{2}{5}$; $rd(x) = 4/9$]

2) $M = F(2,4)$ • $\xi \in M$ con esp $\geq 4 \Rightarrow \xi \in \mathbb{Z}$

• determ $\max\{\xi \in M \mid \xi > 0 \text{ e } \xi \notin \mathbb{Z}\}$

[$\underline{\text{Risp}}$: $\max = 2^3\ 0{,}1111$]

3) $M = F(10,3)$; determ $\{\xi \in M \mid \xi \oplus 1 > 1\}$

[$\underline{\text{Risp}}$: $\xi \geq 10^{-2}\ 0{,}501$]