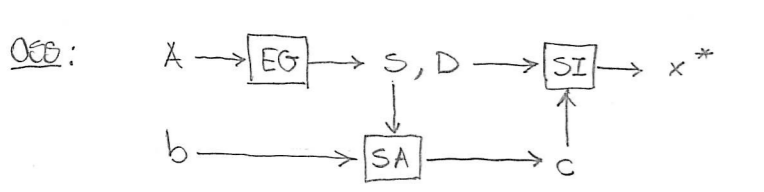


• STUDIO IN F(2,53)



$\epsilon_D = \frac{\|\hat{D} - D\|}{\|D\|}$ ,  $\epsilon_C = \frac{\|\hat{c} - c\|}{\|c\|}$



$\hat{x} = \hat{SI}(\hat{D}, \hat{c})$ ,  $x^* = SI(D, c)$

$\tilde{x} = SI(\hat{D}, \hat{c})$

$\Rightarrow \epsilon_t = \frac{\|\hat{x} - x^*\|}{\|x^*\|} \leq \frac{\|\hat{x} - \tilde{x}\|}{\|\tilde{x}\|} \frac{\|\tilde{x}\|}{\|x^*\|} + \frac{\|\tilde{x} - x^*\|}{\|x^*\|}$   
 $\leftarrow \epsilon_a$   
 $\leq \frac{\|\tilde{x} - x^*\|}{\|x^*\|} + 1$   
 $\leftarrow \epsilon_d$

$\Rightarrow \epsilon_t \leq \epsilon_a + \epsilon_d + \epsilon_a \epsilon_d$

Supponendo  $\epsilon_a$  piccolo...

Teo (condiz, I) + Teo (condiz, II)  $\Rightarrow$

- $\epsilon_d \leq \mu(D) \epsilon_C$  ( $\epsilon_D = 0$ )
- $\hat{\epsilon}_d \leq \mu(D) \epsilon_D$  ( $\epsilon_C = 0$ )

$\Rightarrow$  occorre studiare  $\mu(D)$ !

Es:  $\gamma \in (0, 1)$ ,  $A(\gamma) = \begin{bmatrix} \gamma & 1 \\ 1 & 0 \end{bmatrix}$ ,  $A^{-1}(\gamma) = \begin{bmatrix} 0 & 1 \\ 1 & -\gamma \end{bmatrix}$

•  $\mu_\infty(A(\gamma)) = (1 + \gamma)^2 < 4$  (numero di condiz basso)

• EG:  $(S(\gamma), D(\gamma)) = \left( \begin{bmatrix} 1 & 0 \\ 1/\gamma & 1 \end{bmatrix}, \begin{bmatrix} \gamma & 1 \\ 0 & -1/\gamma \end{bmatrix} \right)$

•  $\mu_\infty(D(\gamma)) \geq \frac{1}{\gamma^2}$  NON LIMITATO per  $\gamma \in (0, 1)$

Obs: EGPP (EG con Pivoting Parziale)

"al passo k, si utilizza come pivot:  $\max_{i \geq k} |a_{ik}^{(k)}|$ "

Es (continua):

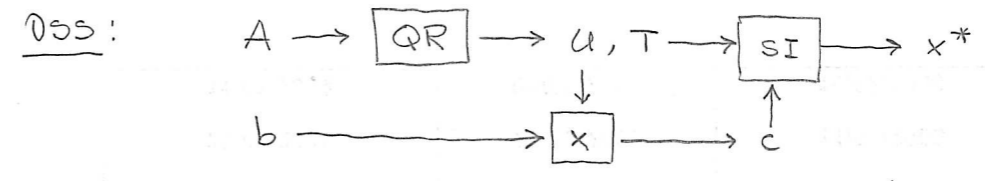
• EGPP:  $(P, S(\gamma), D(\gamma)) = \left( \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ \gamma & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$   
 con  $\mu_\infty(D) = 1$

In generale, con EGPP si ottiene  $\mu(D) \leq m 2^m \mu(A)$

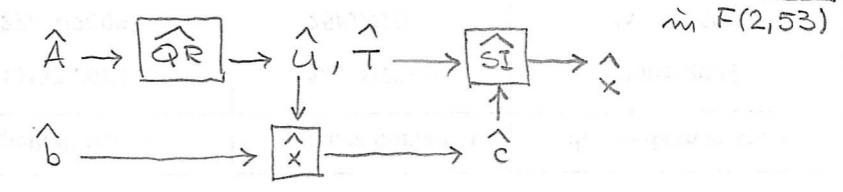
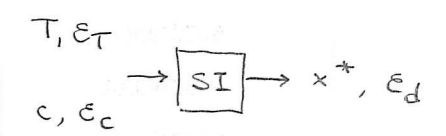
Es (per caso):  $T \in \mathbb{R}^{n \times n}$  tr sup; verif che  $\|T\|_\infty \geq \max_k |t_{kk}|$

e q.d'chi, se T invert, che  $\|T^{-1}\|_\infty \geq \max_k |t_{kk}^{-1}|$ .

$\Rightarrow \mu_\infty(T) \geq \frac{\max |t_{kk}|}{\min |t_{kk}|}$



$\epsilon_T, \epsilon_C \dots$



- Teo condiz  $\Rightarrow$
- $\epsilon_d \leq \mu(T) \epsilon_T$  ( $\epsilon_C = 0$ )
  - $\hat{\epsilon}_d \leq \mu(T) \epsilon_C$  ( $\epsilon_T = 0$ )

$\Rightarrow$  occorre studiare  $\mu(T)$ !

Oss:  $(\mathbb{R}^m, N_2)$  •  $T = U^T A \Rightarrow \|T\|_2 \leq \|A\|_2$

•  $T^{-1} = A^{-1} U \Rightarrow \|T^{-1}\|_2 \leq \|A^{-1}\|_2$

$$\Rightarrow \boxed{\mu_2(T) \leq \mu_2(A)}$$

$$\boxed{\text{Es (per casa): } U \text{ ortogonale} \\ \Rightarrow \|U\|_2 = 1}$$

Oss: sussiste il sep

TEO:  $\hat{x} \in \mathbb{R}^m$  ottenuto dalla procedura

$$(I) (\hat{S}, \hat{D}) = \widehat{EG}(A)$$

$$(II) \hat{c} = \widehat{SA}(\hat{S}, b)$$

$$(III) \hat{x} = \widehat{SI}(\hat{D}, \hat{c})$$

Allora:  $\exists \delta A \in \mathbb{R}^{n \times n}$  t.c.:

$$\bullet \| \delta A \| \leq \mu \|\hat{S}\| \|\hat{D}\| \quad [\text{a precis di macchina}]$$

$$\bullet (A + \delta A) \hat{x} = b$$

OVVERO:  $\hat{x}$  è la soluz CALCOLATA OP IN IR di un sist di eq ottenuto perturbando "poco" la matrice del sist originario.

Oss (int fisica): Se il sist  $Ax = b$  ha origin fisica,

i dati  $A$  e  $b$  sono affetti da errore;

se  $\delta A \approx$  errore di origin fisica, allora

$\hat{x}$  è "fisicam significativa"