- STUDIO IN $F(2,53)$

Oss:

$$\begin{tikzcd} A \arrow[r] & \boxed{EG} \arrow[r] & S, D \arrow[r] \arrow[d] & \boxed{SI} \arrow[r] & x^* \\ b \arrow[rr] & & \boxed{SA} \arrow[r] & c \arrow[u] & \end{tikzcd}$$

in $\mathbb{R}$

in $F(2,53)$

$$\begin{tikzcd} \hat{A} \arrow[r] & \boxed{\widehat{EG}} \arrow[r] & \hat{S}, \hat{D} \arrow[r] \arrow[d] & \boxed{\widehat{SI}} \arrow[r] & \hat{x} \\ \hat{b} \arrow[rr] & & \boxed{\widehat{SA}} \arrow[r] & \hat{c} \arrow[u] & \end{tikzcd}$$

ad es: $\hat{A} = rd(A)$, $\hat{b} = rd(b)$

$$\varepsilon_D = \frac{\|\hat{D} - D\|}{\|D\|}, \quad \varepsilon_C = \frac{\|\hat{c} - c\|}{\|c\|}$$

$$\begin{matrix} D, \varepsilon_D \\ c, \varepsilon_c \end{matrix} \longrightarrow \boxed{SI} \longrightarrow x^*, \varepsilon_d$$

$\hat{x} = \widehat{SI}(\hat{D}, \hat{c})$, $x^* = SI(D, c)$

$\tilde{x} = SI(\hat{D}, \hat{c})$

$$\Rightarrow \varepsilon_t = \frac{\|\hat{x} - x^*\|}{\|x^*\|} \leq \underbrace{\frac{\|\hat{x} - \tilde{x}\|}{\|\tilde{x}\|}}_{\varepsilon_a} \frac{\|\tilde{x}\|}{\|x^*\|} + \underbrace{\frac{\|\tilde{x} - x^*\|}{\|x^*\|}}_{\varepsilon_d}$$

$$\frac{\|\tilde{x}\|}{\|x^*\|} \leq \frac{\|\tilde{x} - x^*\|}{\|x^*\|} + 1$$

$$\Rightarrow \varepsilon_t \leq \varepsilon_a + \varepsilon_d + \varepsilon_a \varepsilon_d$$

supponendo ε_a piccolo...

> Teo (condiz, I) + Teo (condiz, II) $\Rightarrow$
> - $\varepsilon_d \leq \mu(D)\, \varepsilon_c \quad (\varepsilon_D = 0)$
> - $\hat{\varepsilon}_d \leq \mu(D)\, \varepsilon_D \quad (\varepsilon_c = 0)$
>
> $\Rightarrow$ occorre studiare $\mu(D)$!

Es: $\gamma \in (0,1)$, $A(\gamma) = \begin{bmatrix} \gamma & 1 \\ 1 & 0 \end{bmatrix}$, $A^{-1}(\gamma) = \begin{bmatrix} 0 & 1 \\ 1 & -\gamma \end{bmatrix}$

- $\mu_\infty(A(\gamma)) = (1+\gamma)^2 < 4$ (numero di condiz basso)
- EG: $(S(\gamma), D(\gamma)) = \left(\begin{bmatrix} 1 & 0 \\ 1/\gamma & 1 \end{bmatrix}, \begin{bmatrix} \gamma & 1 \\ 0 & -1/\gamma \end{bmatrix} \right)$
- $\mu_\infty(D(\gamma)) \geq \frac{1}{\gamma^2}$ NON LIMITATO per $\gamma \in (0,1)$

Oss: EGPP (EG con Pivoting Parziale)

"al passo k, si utilizza come pivot: $\max_{i \geq k} |a_{ik}^{(k)}|$ "

> Es (continua):
> - EGPP: $(P, S(\gamma), D(\gamma)) = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ \gamma & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$
>
>   con $\mu_\infty(D) = 1$
>
> In generale, con EGPP si ottiene $\boxed{\mu(D) \leq n 2^n \mu(A)}$

Es (per casa): $T \in \mathbb{R}^{n \times n}$ tr sup; verif che $\|T\|_\infty \geq \max_k |t_{kk}|$ e q.di che, se T invert, che $\|T^{-1}\|_\infty \geq \max_k |t_{kk}^{-1}|$.

$$\Rightarrow \mu_\infty(T) \geq \frac{\max |t_{kk}|}{\min |t_{kk}|}$$

Oss:

$$\begin{tikzcd} A \arrow[r] & \boxed{QR} \arrow[r] & U, T \arrow[r] \arrow[d] & \boxed{SI} \arrow[r] & x^* \\ b \arrow[rr] & & \boxed{x} \arrow[r] & c \arrow[u] & \end{tikzcd}$$

in $\mathbb{R}$

in $F(2,53)$

$$\begin{tikzcd} \hat{A} \arrow[r] & \boxed{\widehat{QR}} \arrow[r] & \hat{U}, \hat{T} \arrow[r] \arrow[d] & \boxed{\widehat{SI}} \arrow[r] & \hat{x} \\ \hat{b} \arrow[rr] & & \boxed{\hat{x}} \arrow[r] & \hat{c} \arrow[u] & \end{tikzcd}$$

$\varepsilon_T, \varepsilon_c \ldots$

$$\begin{matrix} T, \varepsilon_T \\ c, \varepsilon_c \end{matrix} \longrightarrow \boxed{SI} \longrightarrow x^*, \varepsilon_d$$

Teo condiz $\Rightarrow$
- $\varepsilon_d \leq \mu(T)\, \varepsilon_T \quad (\varepsilon_c = 0)$
- $\hat{\varepsilon}_d \leq \mu(T)\, \varepsilon_c \quad (\varepsilon_T = 0)$

$\Rightarrow$ occorre studiare $\mu(T)$!

Oss: $(\mathbb{R}^n, N_2)$ • $T = U^T A \Rightarrow \|T\|_2 \leq \|A\|_2$

• $T^{-1} = A^{-1} U \Rightarrow \|T^{-1}\|_2 \leq \|A^{-1}\|_2$

$\Rightarrow \boxed{\mu_2(T) \leq \mu_2(A)}$

Es (per caso): U ortogonale $\Rightarrow \|U\|_2 = 1$

Oss: sussiste il seg

TEO: $\hat{x} \in \mathbb{R}^n$ ottenuto dalla procedura

(I) $(\hat{S}, \hat{D}) = \widehat{EG}(A)$

(II) $\hat{c} = \widehat{SA}(\hat{S}, b)$

(III) $\hat{x} = \widehat{SI}(\hat{D}, \hat{c})$

Allora: $\exists\ \delta A \in \mathbb{R}^{n \times n}$ t.c.:

- $\|\delta A\| \leq n u \|\hat{S}\| \|\hat{D}\|$ [u precis di macchina]
- $(A + \delta A)\hat{x} = b$

OVVERO: $\hat{x}$ è la soluz CALCOLATA op in $\mathbb{R}$ di un sist di eq ottenuto perturbando "poco" la matrice del sist originario.

Oss (int fisico): se il sist $Ax = b$ ha origine fisica, i dati A e b sono affetti da errore; se $\delta A \approx$ errore di origine fisica, allora $\hat{x}$ è "fisicam significativa"