Oss: Siano $f: \mathbb{R} \to \mathbb{R}$, $\varphi: F(2,53) \to F(2,53)$

t.c. φ stabile $\forall \xi$; $g: \mathbb{R} \to \mathbb{R}$, $\gamma: F(2,53) \to F(2,53)$

t.c. γ stabile $\forall \xi$.

Si utilizza $\varphi(\gamma(x))$ per ottenere $f(g(x))$.

È stabile?

Stabile significa: $|\epsilon_a| = \left| \dfrac{\varphi(\gamma(\xi)) - f(g(\xi))}{f(g(\xi))} \right|$ piccolo,

ovvero $\exists \, \epsilon_a$ piccolo t.c.

$$\varphi(\gamma(\xi)) = (1 + \epsilon_a) \, f(g(\xi)).$$

Ma: $\varphi(\gamma(\xi)) = \varphi\big(\underbrace{(1+\epsilon_1)}_{\text{piccolo}} g(\xi) \big) =$

$= (1 + \epsilon_2) \, f\big(\underbrace{(1+\epsilon_1)}_{\text{piccolo}} g(\xi) \big)$

$= (1 + \epsilon_2)(1 + \epsilon_*) \, f(g(\xi))$

$\underbrace{\qquad}_{\text{è la } f \text{ di condiz di } f \text{ in } g(\xi), \, \epsilon_1}$

q.d.: stabile se il calcolo di f è

ben condiz in $g(\xi)$.

Es: $f(x) = \operatorname{sen} \sqrt{x}$, $\varphi(\xi) = \text{SEN}(\text{SQRT}(\xi))$.

Es: $f(x) = \sqrt{\operatorname{sen} x}$, $\varphi(\xi) = \text{SQRT}(\text{SEN}(\xi))$

• Stabilità: $\varphi(\xi) = \text{SQRT}\big((1+\epsilon_1) \operatorname{sen} \xi \big) =$

$= (1 + \epsilon_2) \sqrt{(1+\epsilon_1) \operatorname{sen} \xi} =$

$= (1 + \epsilon_2)(1 + \epsilon_*) \sqrt{\operatorname{sen} \xi}$

$\underset{\text{piccolo perchè } \sqrt{x} \text{ sempre ben condiz... STABILE } \forall \xi !}{\big\downarrow}$

• condizionamento: $f\big((1+\epsilon)x\big) = \sqrt{\operatorname{sen}((1+\epsilon)x)}$

$= \sqrt{(1 + \epsilon_*) \operatorname{sen} x} = (1 + \epsilon'_*) \sqrt{\operatorname{sen} x}$

$\underset{\text{non necess piccolo: calcolo di } \operatorname{sen} x \text{ può essere non ben condiz!}}{\big\downarrow}$

$\Rightarrow$ non ben condiz $\forall x$.

Es: $f(x) = \sqrt{e^x}$, $\varphi(\xi) = \text{SQRT}(\text{EXP}(\xi))$

Studiare stab e condizionam.

Es: $x = 2/5$;
- esp e froz in base tre;
- $rd(x)$ in $F(3,2)$.

$$\left[\text{Sol}: \quad \frac{2}{5} = 3^0 \cdot \frac{2}{5} = 3^0 \; 0,101\ldots \notin F(3,2) \right.$$
$$\left. rd(x) = 3^0 \; 0,11 = 4/9 \right]$$

Es: $M = F(2,4)$;
- $\xi \in M$ con esp $\geq 4 \Rightarrow \xi \in \mathbb{Z}$
- determ $\max \left\{ \xi \in M \mid \xi > 0 \text{ e } \xi \notin \mathbb{Z} \right\}$

$$\left[\text{Sol}: \quad \max = 2^3 \; 0,1111 \right]$$

Es: $M = F(10,3)$;
- determ $\left\{ \xi \in M \mid \xi \oplus 1 > 1 \right\}$

$$\left[\text{Sol}: \quad \xi \geq 10^{-2} \; 0,501 \right]$$