II) Studio dell'errore algoritmico (STABILITA')

Oss (caso elementare): $f: \mathbb{R}^n \to \mathbb{R}$, $\varphi: F^n(2,53) \to F(2,53)$
t.c. $\forall \xi_1, \dots, \xi_n$: $\varphi(\xi_1,\dots,\xi_n) = rd(f(\xi_1,\dots,\xi_n))$.

Allora:
$$|\epsilon_a| = \left| \frac{\varphi(\xi_1,\dots,\xi_n) - f(\xi_1,\dots,\xi_n)}{f(\xi_1,\dots,\xi_n)} \right| \le u$$

Es (op aritm): $* \in \{+, -, \times, /\}$

$f(x_1, x_2) = x_1 * x_2$, $\varphi(\xi_1, \xi_2) = \xi_1 \circledast \xi_2 = rd(\xi_1 * \xi_2)$ ...

Es (f. elem): $f: \mathbb{R} \to \mathbb{R}$ f. elementare ...

Es (caso non elementare):

(A) $f(x) = \operatorname{sen} x + \cos x$; $\varphi(\xi) = SEN(\xi) \oplus COS(\xi)$

- $SEN(\xi) = rd(\operatorname{sen}\xi) = (1+\epsilon_1)\operatorname{sen}\xi$ con $|\epsilon_1| \le u$
- $COS(\xi) = rd(\cos\xi) = (1+\epsilon_2)\cos\xi$ con $|\epsilon_2| \le u$
- $\eta_1 \oplus \eta_2 = rd(\eta_1 + \eta_2) = (1+\epsilon_3)(\eta_1+\eta_2)$ con $|\epsilon_3| \le u$

$$\Rightarrow \varphi(\xi) = \left((1+\epsilon_1)\operatorname{sen}\xi + (1+\epsilon_2)\cos\xi\right)(1+\epsilon_3)$$
$$= (1+\epsilon_1)(1+\epsilon_3)\operatorname{sen}\xi + (1+\epsilon_2)(1+\epsilon_3)\cos\xi$$
$$= (1+\theta_1)\operatorname{sen}\xi + (1+\theta_2)\cos\xi \quad \text{con } |\theta_1|, |\theta_2| \le 2u + \cancel{u^2}$$

$\Rightarrow \epsilon_a = C(\operatorname{sen}\xi, \cos\xi; \theta_1, \theta_2)$

f di condiz di $x_1 + x_2$.

(B) $f(x) = x(x+1)$; $\varphi(\xi) = \xi \otimes (\xi \oplus 1)$

$\boxed{\text{Oss: } 1 \in F(2,53)}$

$$\Rightarrow \varphi(\xi) = (1+\epsilon_2)\,\xi\,(1+\epsilon_1)(\xi+1) \quad \text{con } |\epsilon_1|, |\epsilon_2| \le u$$
$$= (1+\epsilon_2)(1+\epsilon_1)\,\xi(\xi+1)$$
$$= (1+\theta) f(\xi) \quad \text{con } |\theta| \le 2u + \cancel{u^2}$$

$\Rightarrow \epsilon_a = \theta$

In generale: $f: \mathbb{R}^n \to \mathbb{R}$, $\varphi: F^n(2,53) \to F(2,53)$
e $\varphi(\xi_1,\dots,\xi_n)$ ottenuto calcolando r f. predef

$\Rightarrow \epsilon_a = S(\xi_1,\dots,\xi_n; \epsilon_1,\dots,\epsilon_r)$ con $|\epsilon_k| \le u$

def: $\varphi: F^n(2,53) \to F(2,53)$ ALGORITMO STABILE per il calcolo di $f: \mathbb{R}^n \to \mathbb{R}$ in $\xi_1,\dots,\xi_n \in F(2,53)$ SE:

$\forall\, \epsilon_1,\dots,\epsilon_r$ t.c. $|\epsilon_1|,\dots,|\epsilon_r| \le u$ si ha
$$|S(\xi_1,\dots,\xi_n; \epsilon_1,\dots,\epsilon_r)| \le ku$$
con k intero "non troppo grande".

Es:
- caso elementare: sempre stabile
- A) stabile se $\operatorname{sen}\xi + \cos\xi$ ben condiz
- B) sempre stabile