Es (formule per il calcolo):

- $N_1(A) = \max\{N_1(a_1), \dots, N_1(a_n)\} = \|A\|_1$ | $N_2(A) = \dots$
- $N_\infty(A) = N_1(A^T) = \|A\|_\infty$
- $A = \begin{pmatrix} 8 & 0 & -2 \\ -1 & 1 & 1 \\ 6 & 3 & 4 \end{pmatrix}$; $\|A\|_1 = 15$, $\|A\|_\infty = 13$

PROPRIETÀ

- $N(Av) \le \|A\|_N \, N(v)$ (dim ...)
- $\|AB\|_N \le \|A\|_N \, \|B\|_N$ (Es: dimostrare!)

- (int geom): $\|A\|_N = \max\{N(Av),\ N(v) = 1\}$

  se A invert: $\|A^{-1}\|_N = \left(\min\{N(Av),\ N(v)=1\}\right)^{-1}$

Es:

- $A \in \mathbb{R}^{n\times n}$; dim che $\|A\|_1 \ge \max |a_{ij}|$
- $A \in \mathbb{R}^{n\times n}$ invert; dim che $\|A\|_N \, \|A^{-1}\|_N \ge 1$ ($\Rightarrow \|A\| \ne 0$)
- $A \in \mathbb{R}^{n\times n}$ invert, $b \in \mathbb{R}^n$, x^* t.c. $Ax^* = b$; dim che

$$\frac{N(b)}{\|A\|_N} \le N(x^*) \le \|A^{-1}\|_N \, N(b)$$

Condizionamento

- dati: $A \in \mathbb{R}^{n\times n}$, invert; $b \in \mathbb{R}^n$

* soluz: $x^* = A^{-1} b$

- dati perturbati: $A + \delta A \in \mathbb{R}^{n\times n}$ invert; $b + \delta b \in \mathbb{R}^n$ — perturbaz sui dati

* soluz del Pb perturbato: $\hat{x}$

def: $(\mathbb{R}^n, N)$, errore relativo sui dati: $\dfrac{\delta A}{\|A\|_N}$, $\dfrac{\delta b}{\|b\|_N}$;

err relativo trasm dai dati: $\dfrac{\hat{x} - x^*}{\|x^*\|_N}$, $\dfrac{\hat{x} - x^*}{\|\hat{x}\|_N}$

FUNZIONE di CONDIZIONAMENTO:

$$\frac{\hat{x} - x^*}{\|x^*\|_N} = F\left(A, b; \frac{\delta A}{\|A\|_N}, \frac{\delta b}{\|b\|_N}\right)$$

Es: $\gamma \in (0,1)$, $A(\gamma) = \begin{bmatrix} \gamma & 0 \\ 0 & 1/\gamma \end{bmatrix}$, $b = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$; $\delta b = \begin{bmatrix} \delta b_1 \\ \delta b_2 \end{bmatrix}$, $\delta A = 0$

- $\det A(\gamma) = 1$
- soluz di $A(\gamma)x = b$: $x^* = \begin{bmatrix} b_1/\gamma \\ \gamma b_2 \end{bmatrix}$
- soluz sist perturbato $A(\gamma)x = b + \delta b$: $\hat{x} = \begin{bmatrix} (b_1 + \delta b_1)/\gamma \\ \gamma(b_2 + \delta b_2) \end{bmatrix}$

I caso: $b_1 \ne 0$, $b_2 = 0$ $\Rightarrow$

$$\frac{\hat{x} - x^*}{\|x^*\|_\infty} = \begin{bmatrix} \delta b_1/\gamma \\ \gamma \delta b_2 \end{bmatrix} \frac{\gamma}{|b_1|} = \frac{\begin{bmatrix} \delta b_1 \\ \gamma^2 \delta b_2 \end{bmatrix}}{\|b\|_\infty} \quad \text{(f. di condizion.)}$$

POSTO $\epsilon_d = \dfrac{\|\hat{x} - x^*\|_\infty}{\|x^*\|_\infty}$, $\epsilon_b = \dfrac{\|\delta b\|_\infty}{\|b\|_\infty}$: $\epsilon_d \le \epsilon_b$

Pb. "ben condizionato": $(\forall \delta b,\ \epsilon_d \le \epsilon_b)$

II caso: $b_1 = 0$, $b_2 \ne 0$, $\delta b_2 = 0$ $\Rightarrow$

$$\frac{\hat{x} - x^*}{\|x^*\|_\infty} = \frac{1}{\gamma |b_2|} \begin{bmatrix} \delta b_1/\gamma \\ 0 \end{bmatrix} = \frac{1}{\gamma^2} \frac{\delta b}{\|b\|_\infty} \quad \text{(f. di cond.)}$$

... $\epsilon_d = \dfrac{1}{\gamma^2} \epsilon_b$

Pb. "mal condizionato": $(\exists\ \delta b :\ \epsilon_d \gg \epsilon_b)$