Es (continuazione): $h(x) = \frac{\cos x}{2}$ • $\exists$ pu in $[0,\pi/2]$

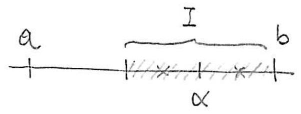
- $|h'(x)| \leq 1/2$ in $[0,\pi/2]$ ($\Rightarrow$ unicita' p.u)
- $\forall c \in [0,\pi/2]$ la succes converge al p.u
- costruz. grafica ...

> Oss: se $[a,b]$ ed h verificano ip. (1) e (2), non e' vero che $\forall x \in [a,b]$ si ha $h(x) \in [a,b]$!

Es: $h: [1,7] \to \mathbb{R}$ t.c. $h(x) = 3 - \frac{x}{2}$;

- 2 e' p.u ; $|h'(x)| \leq 1/2$ su $[1,7]$ MA $h(6) \notin [1,7]$!

Oss: se $[a,b]$ ed h verificano ip (1) e (2) allora:



$r = \min\{|\alpha - a|, |b - \alpha|\}$, $I = [\alpha - r, \alpha + r]$

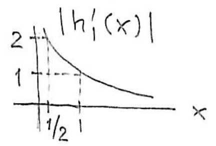
$\forall x \in I$, $h(x) \in I$ (infatti...)

Q.d:

> SE $[a,b]$, h verif ip (1) e (2)
> ALLORA $c =$ (l'estremo di $[a,b]$ piu' vicino al p.u)
> genera succes che verifica ip (3) !

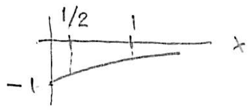
Es: $f(x) = x + \log x$; $h_1(x) = -\log x$, $h_2(x) = e^{-x}$, $h_3(x) = \frac{x + e^{-x}}{2}$.

- $f: (0,+\infty) \to \mathbb{R}$, $f'(x) = 1 + \frac{1}{x} > 0 \Rightarrow$ # zeri ≤ 1
  - * $\lim_{x\to 0} f(x) = -\infty$, $f(1) = 1 \Rightarrow \exists!$ zero, $\alpha \in (0,1]$
  - * $f(1/2) = 1/2 - \log 2 < 0 \Rightarrow \alpha \in [1/2, 1]$
- $h_1'(x) = -\frac{1}{x}$;



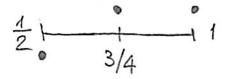
$\Rightarrow |h'(\alpha)| > 1$ e q.di $\nexists [a,b]$ che verif. ip (1), (2).

- $h_2'(x) = -e^{-x}$



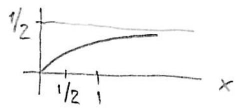
, $|h_2'(x)| \leq e^{-1/2} = L_2 < 1$ su $[1/2, 1]$

* $[1/2, 1]$, h_2 verif ip (1) e (2) ;



$\Rightarrow$ l'estr piu' vicino e' $c = 1/2$.

- $h_3'(x) = \frac{1 - e^{-x}}{2}$



, $|h_3'(x)| \leq \frac{1 - 1/e}{2} = L_3 < 1$ su $[1/2, 1]$

* $[1/2, 1]$, h_3 verif. ip (1) e (2) ; $c = 1/2$ rende verif (3).

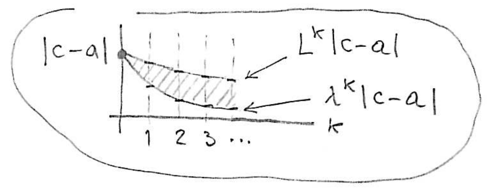
Q.di: h_2, h_3 ok, h_1 ?

> Oss: se $h \in C^1$ e α pu di h tali che $|h'(\alpha)| > 1$ allora:
> $\exists \hat{k}$ t.c. $x_k = \alpha$ per $k \geq \hat{k}$ OPPURE $x_k \not\to \alpha$

Oss (rapidita' di convergenza): se h, $[a,b]$, c verif ip teo conv loc e $x_k \neq \alpha$ per ogni k, allora:

- SE $0 < \lambda \leq |h'(x)| \leq L < 1$ su $[a,b]$, ALLORA $\lambda^k |c - a| \leq |x_k - \alpha| \leq L^k |c - a|$

ovvero: $|x_k - \alpha| \to 0$...



* ... almeno rapidam come $L^k |c-a|$ ma...
* ... non piu' rapidam di $\lambda^k |c-a|$.

- SE $h'(\alpha) = 0$, $\forall \theta > 0$ si ha: $\lim_{k\to\infty} \frac{|x_k - \alpha|}{\theta^k} = 0$

ovvero: $|x_k - \alpha| \to 0$...

* ... piu' rapidamente di qualsiasi esponenziale

> def (ordine di convergenza DEL METODO):
> - $h \in C^1$, α p.u e $0 < |h'(\alpha)| < 1$: ORDINE di CONVERGENZA ad α : 1
> - $h \in C^2$, α p.u e $h'(\alpha) = 0$, $h''(\alpha) \neq 0$: O.d.C. ad α : 2

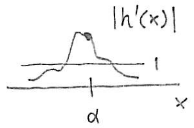
Es (continuazione):

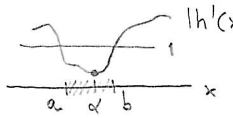
- $1/e \leq |h_2'(x)| \leq 1/\sqrt{e}$
- $\frac{1 - 1/\sqrt{e}}{2} \leq |h_3'(x)| \leq \frac{1 - 1/e}{2}$

OdC ad α : 1 per entrambi, MA siccome $\frac{1 - 1/e}{2} < \frac{1}{e}$ ...

... il metodo def da h_3 garantisce, nel caso peggiore ("piu' lento") una rapidita' di convergenza maggiore del metodo def da h_2.

Oss: $h \in C^1$ ed α p.u di h ; $\exists\, [a,b]$ che verifica ip (1) e (2) del Teo di conv loc $\iff |h'(\alpha)| < 1$.

(dim : 

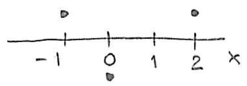
; 

siccome $h \in C^1$, h' e' continua ...)

Es: Sia $h(x) = e^x - x - 2 : \mathbb{R} \to \mathbb{R}$;

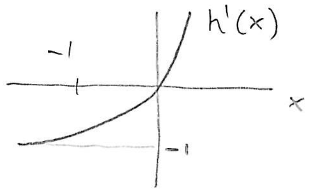
- SEPARARE i p.u di h
- per ciascun p.u, discutere l'uso del met it (operando in $\mathbb{R}$)

Sol: • $h(x) = x \iff \underbrace{e^x - 2(x+1)}_{F(x)} = 0$; $F \in C^\infty(\mathbb{R})$, $F''(x) > 0 \;\; \forall x \in \mathbb{R} \Rightarrow \#\text{zeri} \leq 2$

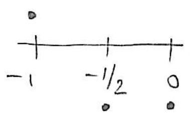


$\Rightarrow \#\text{zeri} = 2$; $\boxed{\alpha_1 \in (-1,0) \quad \alpha_2 \in (0,2)}$

• $h'(x) = e^x - 1$



* $-1 < h'(\alpha_1) < 0 \Rightarrow$ metodo it utilizz per appross α_1 ; OdC ad α_1 : 1 ;



$\Rightarrow$ $c = -1$ genera succes convergente (siccome $h'(x) < 0$, la succes...)

* l'intervallo $[0,2]$ non verif ip (2)...

$\Rightarrow [1,2] \ni \alpha_2$, MA $h'(x) > 1 \;\; \forall x \in [1,2] \Rightarrow h'(\alpha_2) > 1$

q.di : metodo it non utilizz per appross α_2.