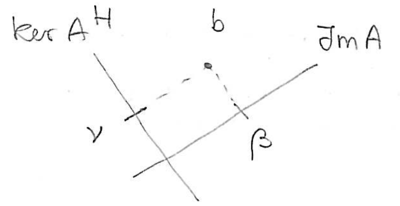
Calcolo pseudoinv, <u>CASI PARTICOLARI</u> :

(I) $A \in \mathbb{C}^{n\times k}$ a <u>colonne lin indip</u> , $b \in \mathbb{C}^n$

- $\ker A = \{0\} \Rightarrow \mathcal{S}_{MQ}(A,b)$ ha un solo elem : $x_* = A^+ b$
- $\mathcal{S}_{MQ}(A,b) = \mathcal{S}(A,\beta)$ con β = pr ort di b su $\operatorname{Im} A$



x^* è t.c. $Ax^* = \beta$ ovvero

$Ax^* - b \perp \operatorname{Im} A$

e q.di : $A^H(Ax^* - b) = 0$

o $A^H A x^* = A^H b$

- $A^H A$ risulta invertibile (è in $\mathbb{C}^{k\times k}$)

e q.di : $x^* = \underbrace{(A^H A)^{-1} A^H}_{A^+} b$

Es : $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$, ... $A^+ = \begin{bmatrix} 1/2 & 0 & 1/2 \\ 0 & 1 & 0 \end{bmatrix}$

(II) $A \in \mathbb{C}^{n\times k}$ a <u>righe lin indip</u> , $b \in \mathbb{C}^n$

in questo caso : $AA^H \in \mathbb{C}^{n\times n}$ è invert

e risulta $x_* = \underbrace{A^H (AA^H)^{-1}}_{A^+} b$

Es : $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, ... $A^+ = \begin{bmatrix} 0 & 1/2 \\ 1 & 0 \\ 0 & 1/2 \end{bmatrix}$

Es : $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, $c = \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix}$

determ $\mathcal{S}(A,b)$, $\mathcal{S}_{MQ}(A,b)$, $\mathcal{S}(A,c)$ e $\mathcal{S}_{MQ}(A,c)$

Es : $A = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

determ $\mathcal{S}(A,b)$ e $\operatorname{Im} A^T \cap \mathcal{S}(A,b)$

Es : $A = \begin{bmatrix} 3 & 0 \\ 0 & -2 \end{bmatrix}$ determ decomp ai v.s.

Es : $R = (1,1) \in \mathbb{R}^{1\times 2}$

determ R^+ e decomp v.s. di R

Es : $A = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $b = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$

determ A^+ e la soluz di $Ax=b$ nel senso dei m q

Es : $A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 0 & 2 \\ 1 & 0 & 2 \end{bmatrix} \in \mathbb{C}^{3\times 3}$ det FCJ(A) e matr che realizza la sim.

Es : $A = \begin{bmatrix} 3 & 0 & 1 \\ 3 & 0 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$

determ
- $\operatorname{im} A$, $\ker A^T$, $\operatorname{im} A^T$, $\ker A$
- $\mathcal{S}_{MQ}(A,b)$

Es: $U=(u_1,u_2)\in\mathbb{R}^{2\times 2}$, $\Sigma=\begin{pmatrix}2&0&0\\0&0&0\end{pmatrix}$, $V=(v_1,v_2,v_3)\in\mathbb{R}^{3\times 3}$

decomp ai v s di $M\in\mathbb{R}^{2\times 3}$.

- indicare basi o n di $\ker M$ e $\operatorname{Im} M$
- determ l'ins delle soluz di $Mx=u_1+u_2$ nel senso dei mq
