

$$\begin{aligned}
 & \int_1^{+\infty} \sqrt{x} - A\sqrt{x+1} - B\sqrt{x-1} \, dx = \\
 &= \int_1^{+\infty} \sqrt{x} \left[1 - A\sqrt{1 + \frac{1}{x}} - B\sqrt{1 - \frac{1}{x}} \right] \, dx = \\
 &= \int_1^{+\infty} \sqrt{x} \left[1 - A - B - \frac{1}{2}A\frac{1}{x} + \frac{1}{2}B\frac{1}{x} + \right. \\
 &\quad \left. \underbrace{\frac{1}{8}\frac{A}{x^2} + \frac{1}{8}\frac{B}{x^2} + o(\frac{1}{x^2})} \right] \, dx \\
 & \left. \begin{array}{l} \sqrt{1+t} = 1 + \frac{1}{2}t - \frac{1}{8}t^2 + o(t^2) \\ f(0) = 1 \\ f'(t) = \frac{1}{2\sqrt{1+t}} \Big|_{t=0} = \frac{1}{2} \\ f''(t) = \left[\frac{1}{2}(1+t)^{-\frac{1}{2}} \right]' = \frac{1}{2} \left(-\frac{1}{2} \right) (1+t)^{-\frac{3}{2}} \\ \text{and for } t=0 \text{ we have} \\ -\frac{1}{4} \end{array} \right. \\
 & \left\{ \begin{array}{l} A+B=1 \\ A-B=0 \quad A=B=\frac{1}{2} \end{array} \right. //
 \end{aligned}$$

$$A = B = \frac{1}{2}$$

$\int_1^{+\infty} \sqrt{x} \left[\frac{1}{8} \frac{1}{x^2} + o\left(\frac{1}{x^2}\right) \right]$

$$\approx \frac{1}{x^{3/2}} > 1$$

$\int_1^{+\infty} \sqrt{x} \frac{o\left(\frac{1}{x^2}\right)}{\frac{1}{x^2}} \cdot \frac{1}{x^2}$

~~not well~~

$2 > 1$

$\int_1^{+\infty} \frac{\sum \sqrt{x}}{x^2} dx =$

$= \sum \int_1^{+\infty} \frac{1}{x^{3/2}}$

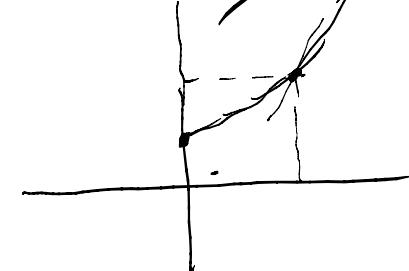
$$\begin{cases} \dot{x} = x \\ x(0) = 1 \end{cases}$$

Probleme d'Cauchy

Condition initiale

$$(x(t_0) = u_0)$$

x grot
de x = grot



$$x \neq 0$$

$$\frac{\dot{x}}{x} = 1$$

$$\int_0^t \frac{\dot{x}(s)}{x(s)} ds = \int_0^t 1 = t$$

$$x(s) = u$$

$$\dot{x}(s) ds = du$$

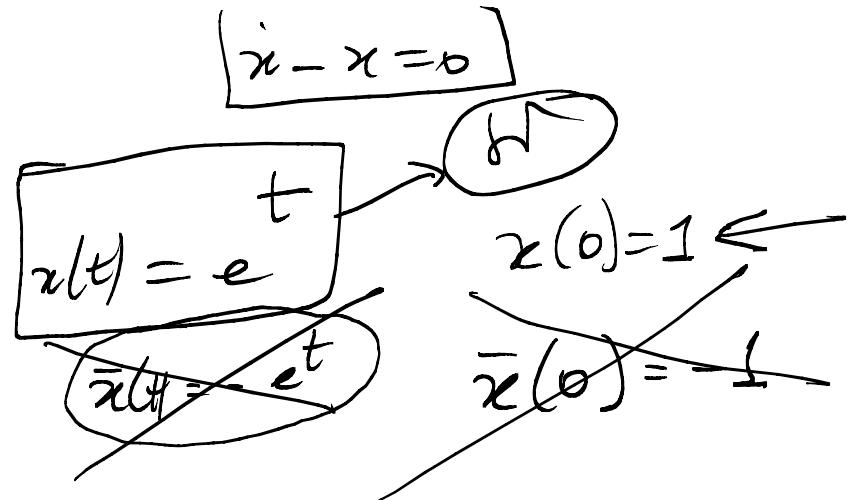
$$\int_{x(0)}^{x(t)} \frac{du}{u} = t$$

$$x(0) = 1$$

$$\Leftrightarrow \log |u(s)| \Big|_1^{x(t)} = t$$

$$\log|x(t)| - \cancel{\log|1|} = t$$

$$\rightarrow |x(t)| = e^t$$



$$\begin{cases} u' = \underbrace{(1+u^2)}_{f(u)} \sin t \\ u(0) = 1 \end{cases}$$

$$1+u^2 > 0 \quad \forall u \in \mathbb{R}$$

$$\frac{u'}{1+u^2} = \sin t$$

$$\Rightarrow \int_0^x \frac{u'(t) dt}{1+u^2(t)} = \int_0^x \sin t dt = -\cos x + 1$$

$$\Rightarrow \int_{u(0)}^{u(x)} \frac{dv}{1+v^2} = \arctg u(x) - \arctg 1$$

$$\arctg u(x) = \arctg \frac{1 - \cos x + \frac{1}{4}}{\frac{u}{4}}$$

$u(x) = \lg \left(\frac{1 + \frac{u}{4}}{1 - \frac{u}{4}} - \cos x \right)$

Se non c'è la condizione iniziale

$$\frac{w}{1+u^2} = \sin t \Rightarrow \int \frac{u'}{1+u^2} dt = \int \sin t dt$$

$$\arctg u = -\cos t + C$$

$$\lg(\arctg u) = \lg(-\cos t + C)$$

$u(t) = \lg(C - \cos t) \quad C \in \mathbb{R}$

$\dot{x} = f$

EQUAZIONI LINEARI

$$\boxed{\dot{x} + \underline{a(t)x} = f(t)}$$

1) EQ. OMogenea ASSOCATA

$$\dot{x} + a(t)x = 0$$

$$\dot{x} = -a(t)x$$

$x \neq 0$

$$\frac{\dot{x}}{x} = -a(t)$$

$$\ln|x| = - \int a(t) dt + C$$

$$x(t) = e^{- \int a(t) dt + C}$$

$$C = e^C \boxed{C e^{- \int a(t) dt}}$$

$$\dot{x} + \alpha(t)x = f(t)$$

$$\left[x e^{\int \alpha(t) dt} + \alpha(t)x e^{\int \alpha(t) dt} \right] = e^{\int \alpha(t) dt} f(t)$$

$$\left(x e^{\int \alpha(t) dt} \right)' = e^{\int \alpha(t) dt} f(t) \quad \text{NOTA}$$

$$x e^{\int \alpha(t) dt} = \int f(t) e^{\int \alpha(t) dt} + C$$

$$\bar{x}(t) = e^{-\int \alpha(t) dt} \left[\int f(t) e^{\int \alpha(t) dt} + C \right]$$

La soluzione generale:

$$x(t) = e^{-\int \alpha(t) dt} \left[\text{tutte c.s.d. omogenee} + C e^{-\int \alpha(t) dt} \int f(t) e^{\int \alpha(t) dt} \right]$$

UNA SOL. E.Q. COMPLETA

$$\dot{x} + tx = t$$

$$x e^{\frac{t^2}{2}} + t x e^{\frac{t^2}{2}} = t e^{\frac{t^2}{2}}$$

$\stackrel{\text{d}u = t \text{ dt}}{\boxed{\quad}}$

$$(x e^{\frac{t^2}{2}})' = t e^{\frac{t^2}{2}}$$

$$x e^{\frac{t^2}{2}} = e^{\frac{t^2}{2}} + c$$

$$e^{kt} = t$$

$$\stackrel{\text{d}u = t \text{ dt}}{\boxed{\quad}} = e^{\frac{t^2}{2}}$$

$$\begin{aligned} \int t e^{\frac{t^2}{2}} \frac{t^2}{2} dt &= u \\ \frac{t^3}{2} dt &= t dt \\ \int e^u du &= e^u = \\ &= e \end{aligned}$$

$$\bar{k} \equiv 1$$

so for unstabile partie,

$$\boxed{\dot{x} = -tx}$$

$x \neq 0$

$$\boxed{\frac{\dot{x}}{x} = -t}$$

$x \equiv 0$ ist stabile

$$\ln|x| = -\frac{t^2}{2} + C$$

$$|x| = e^C e^{-\frac{t^2}{2}} \equiv C e^{-\frac{t^2}{2}}$$

per $C = 0$

elsewhere
to unstabile
 $x \equiv 0$
 $\dot{x} + tx = 0$

$$\boxed{x(t) = C e^{-\frac{t^2}{2}} + 1}$$

$$\overline{x^2} \overline{y''(x)} + \overline{2x} \overline{y'(x)} + \overline{y(x)} = 0$$

$$\boxed{-e^{2t} y''(e^t)} + 2e^t y'(e^t) + y(e^t) = 0$$

$$x = e^t \leftarrow$$

$$t = \lg x$$

$$2(y(e^t))' + \underline{y(e^t)} = 0$$

$$(y(e^t))' = \underline{y'(e^t) e^t}$$

$$\boxed{y(x) = y(e^t) = u(t)}$$

$$(y(e^t))'' = (y'(e^t) e^t)' =$$

$$= y''(e^t) e^t \cdot e^t +$$

$$+ \boxed{y'(e^t) e^t}$$

$$u''(t) + u'(t) + u(t) = 0$$

$$\alpha e^{\lambda_1 t} + \beta e^{\lambda_2 t} = u(t)$$

$$y(x) = \alpha e^{\lambda_1 \lg x} + \beta e^{\lambda_2 \lg x} = \alpha (e^{\lg x})^{\lambda_1} + \beta (e^{\lg x})^{\lambda_2} = \alpha x^{\lambda_1} + \beta x^{\lambda_2}$$

$$\boxed{e^{2t} y''(e^t)} = (y(e^t))'' - (y(e^t))'$$

$$(y(e^t))'' - (y(e^t))' =$$

$$xy'' + 2y' + xy = 0$$

$$\rightarrow \boxed{y(x) = \frac{u(x)}{x}}$$

$$\begin{aligned} u'' - 2\frac{u'}{x} + 2\frac{u}{x^2} + \\ + 2\frac{u'}{x} - 2\frac{u}{x^2} + u = 0 \end{aligned}$$

$$\boxed{u'' + u = 0}$$

$$x \cos x + \beta \sin x \Rightarrow y(x) = x \frac{\cos x}{x} + P \frac{\sin x}{x}$$

$$y' = \frac{xu' - u}{x^2} = \frac{u'}{x} - \frac{u}{x^2}$$

$$y'' = \frac{2u'' - u'}{x^2} - \frac{x^3u' - 2xu}{x^4} =$$

$$= \frac{u''}{x} - \frac{u'}{x^2} - \frac{u}{x^2} + \frac{2u}{x^3}$$

$$= \frac{u''}{x} - 2\frac{u'}{x^2} + 2\frac{u}{x^3}$$

$$u'' + u = e^t \cos t = e^t \frac{1}{2} [e^{it} + e^{-it}] =$$

$$= \frac{1}{2} e^{(1+i)t} + \frac{1}{2} e^{(1-i)t}$$

$f_1(t)$ $\lambda = 1+i$ $f_2(t)$ $\lambda = 1-i$

$$\lambda^2 + 1 = 0 \quad \lambda = i \text{ or } -i$$

- \hat{P}_1, \hat{P}_2
some of
grade 0

une autre partie sera tout de

$$\alpha e^{(1+i)t} + \beta e^{(1-i)t} =$$

$$q_i(t) \stackrel{d}{\longrightarrow} \text{grad } \phi = \text{grad } f_i$$

$$N_p = e^t (\gamma \cos t + \delta \sin t)$$