Note Title 12/14/2021

f integrabile su $[a, c]$ allora

$$\int_a^{+\infty} f \equiv \lim_{c \to +\infty} \int_a^c f$$

$f \geq 0 \Rightarrow c \to \int_a^c f$ crescente

f si dice ASSOLUTAMENTE INTEGRABILE se $\int_a^{+\infty} |f|$ è convergente.

Th. Se f è Ass. Conv. allora è Conv.

f è integrabile in $[a, +\infty[$ se la funzione $c \to \int_a^c f$ è convergente a $+\infty$.

La relativa condizione di Cauchy dice

CNS perché $c \to \int_a^c f$ sia convergente è che

$\forall \varepsilon > 0 \ \exists \bar{x} : \forall c, c' > \bar{x}$ $\left| \int_a^c f - \int_a^{c'} f \right| < \varepsilon$

$+ \int_{c'}^{a}$

$\left| \int_{c'}^{c} f \right| < \varepsilon$

$c' < c$ $\left| \int_{c'}^{c} f \right| \overset{\text{dis. triang.}}{\leq} \int_{c'}^{c} |f| < \varepsilon \quad \forall c, c' > \bar{x}$

perché f è assol. integrabile.

$$\int_1^{+\infty} \frac{1}{x^\alpha}\,dx \quad \begin{cases} \text{esiste (ed è converg)} & \text{se } \alpha > 1 \\ \text{diverge} & \text{se } \alpha \le 1 \end{cases}$$

$$\int_{\frac{3}{2}}^{+\infty} \frac{1}{x \lg^\alpha x}\,dx = \begin{cases} \text{esiste} & \text{se } \alpha > 1 \\ \text{non esiste} & \text{se } \alpha \le 1 \end{cases}$$

$$\int_{\frac{3}{2}}^{c} \frac{1}{x \lg^\alpha x}\,dx \underset{\sim}{\sim} \int_{\frac{3}{2}}^{c} \frac{(\lg x)'}{x \lg^\alpha x}\,dx \overset{y=\lg x}{=\!=} \int_{\lg \frac{3}{2}}^{\lg c} \frac{1}{y^\alpha}\,dy$$

$dy = \frac{1}{x}\,dx$

$$\int \frac{1}{x \lg x (\lg \lg x)^\alpha}$$

$y = f(x)$

$$dy = \frac{dy}{dx}\,dx$$

$$\int_{\pi}^{+\infty} \frac{\sin x}{x}\,dx \quad \begin{cases} \text{converge} \\ \text{non converge assolutamente} \end{cases}$$

$$\int_{\pi}^{c} \frac{\sin x}{x}\,dx = \frac{1}{x}(-\cos x)\Big|_{\pi}^{c} - \int_{\pi}^{c} (-\cos x)\left(-\frac{1}{x^2}\right)dx =$$

$$= \underbrace{\frac{-\cos c}{c}}_{\to 0} - \frac{-\cos\pi}{\pi} - \int_{\pi}^{c} \frac{\cos x}{x^2}\,dx$$

tende ad un limite finito perché $\left|\frac{\cos x}{x^2}\right| \le \frac{1}{x^2}$

l'ultimo integrale esiste perché $\exists \lim_{c\to+\infty} \int_{\pi}^{c} \frac{\cos x}{x^2}\,dx$ del teorema del confronto

Si ha poi $\int_{\pi}^{+\infty} \frac{|\sin x|}{x} = +\infty$

su $[\pi, +\infty[$ $|x| = x$

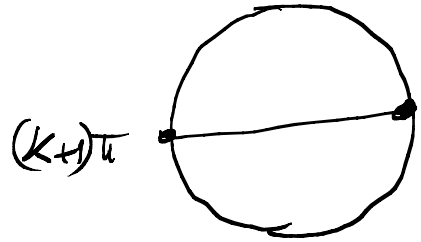
$x \to |\sin x|$

$$\int_{\pi}^{n\pi} \frac{|\sin x|}{x} = \sum_{k=1}^{n-1} \int_{k\pi}^{(k+1)\pi} \frac{|\sin x|}{x} \geq \sum_{k=1}^{n-1} \int_{k\pi}^{(k+1)\pi} \frac{|\sin x|}{(k+1)\pi} = \sum_{k=1}^{n-1} \frac{1}{(k+1)\pi} \int_{k\pi}^{(k+1)\pi} |\sin x|$$

se k è pari $\int_{k\pi}^{(k+1)\pi} |\sin x| = \int_{k\pi}^{(k+1)\pi} \sin x = -\cos x \Big|_{k\pi}^{(k+1)\pi} =$

$= -\cos(k+1)\pi + \cos k\pi$

$-1 \qquad 1$



$= 2$

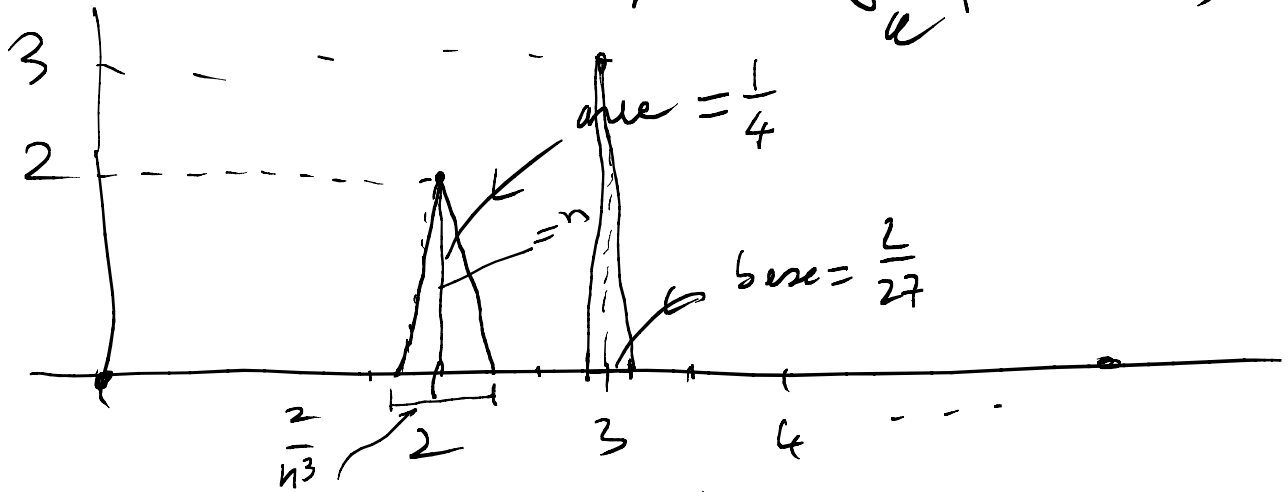
$$\int_{\pi}^{n\pi} \frac{|\sin x|}{x} dx \geq \frac{2}{\pi} \sum_{1}^{n-1} \frac{1}{k+1} \xrightarrow{n \to +\infty} +\infty$$

La funzione $C \to \int_{\pi}^{C} \frac{|\sin x|}{x}\,dx = F(C)$ converge o diverge se $C \to +\infty$

≥ 0

diverge perché $F(n\pi) \to +\infty$

E' FALSO CHE $\int_a^{+\infty} f \Rightarrow f \to 0$ $\quad (x \to +\infty)$



$f(n) = n$

Scelto una serie a termini >0 convergente $\sum \frac{1}{n^2}$

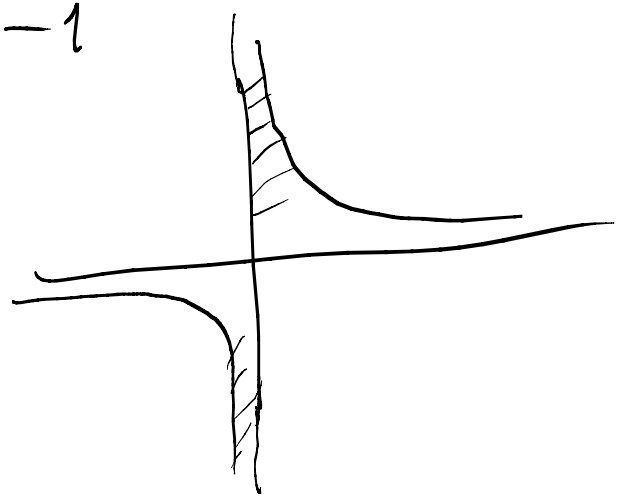
$$\int_0^1 \frac{1}{x}\,dx \equiv \lim_{c\to 0^+} \int_c^1 \frac{1}{x}\,dx = \lim_{c\to 0^+} \log|x|\Big|_c^1 = \log|1| - \log|c| \quad (c>0)$$

$$= -\log c$$

in $[0, x_1]$ f non è limitata inf.

$$= +\infty$$

$$\int_{-1}^0 \frac{1}{x}\,dx = \lim_{c\to 0^-} \int_{-1}^c \frac{1}{x}\,dx = \lim_{c\to 0^-} \log|x|\Big|_{-1}^c = -\infty$$



$$\int_{-1}^1 \frac{1}{x}\,dx \text{ "}=\text{"} \log|1| - \log|-1|$$

$$= 0$$

2 INTEGRALI INDIPENDENTI

$$\int_{-1}^{1} \frac{1}{\sqrt{|x|}}\,dx = \int_{-1}^{0} \frac{1}{\sqrt{|x|}}\,dx + \int_{0}^{1} \frac{1}{\sqrt{|x|}}\,dx$$

$$\int_{0}^{1} \frac{1}{\sqrt{|x|}}\,dx = \int_{c}^{1} \frac{1}{\sqrt{x}} = 2\int_{c}^{1} \frac{1}{2\sqrt{x}} = 2\sqrt{x}\Big|_{c}^{1} =$$

$c \to 0^+$

$c > 0$

$$= 2\sqrt{1} - 2\sqrt{c} =$$

$c \to 0$

$$= 2$$

$\int_{-1}^{0} \frac{1}{\sqrt{|x|}}$ e $\int_{0}^{1} \frac{1}{\sqrt{|x|}}$ entrambe finite, così anche la loro somma $\int_{-1}^{1} \frac{1}{\sqrt{|x|}}$

$$\int_{-1}^{1} \frac{1}{x^2}\,dx = \int_{-1}^{0} + \int_{0}^{1}$$

$$\int_0^1 \frac{1}{x^2} = \lim_{c\to 0^+} \int_c^1 \frac{1}{x^2}\,dx = \lim_{c\to 0^+} -\frac{1}{x}\Big|_c^1 =$$

$$= -1 + \left(\frac{1}{c}\right) \xrightarrow{c\to 0^+} +\infty$$

$$\int_0^1 \frac{1}{x^\alpha}\,dx \quad \begin{cases} \text{esiste finito se } 0<\alpha<1 \\ \text{non esiste se } \alpha \geq 1 \end{cases}$$

Lo stesso per $\int_0^{1/2} \frac{1}{x \log^\alpha x}$

$$\frac{1}{|x-c|^\alpha} \qquad \frac{1}{|x-c|\log^\alpha |x-c|}$$

$$\int_0^1 \log x \, dx$$

Studiare $\int_0^1 -\log x \, dx$

$$\frac{-\log x}{\frac{1}{x}} \underset{x\to 0}{=\!=} \frac{-\frac{1}{x}}{-\frac{1}{x^2}} = x \to 0$$

NESSUNA INFORMAZIONE

$$\lim_{x\to 0^+} \frac{-\lg x}{\frac{1}{\sqrt{x}}} \overset{H}{=} \lim \frac{-\frac{1}{x}}{-\frac{1}{2}x^{-\frac{3}{2}}} = \lim_{0^+} \frac{\frac{1}{x}}{\frac{1}{\sqrt{x^3}}} =$$

$$= \lim_{0^+} \frac{\sqrt{x^3}}{x} = \lim_{0^+} \sqrt{x} = 0 \Rightarrow$$

$\forall \varepsilon > 0 \ \exists \delta :$
$0 < x < \delta$
$\frac{-\lg x}{\frac{1}{\sqrt{x}}} < \varepsilon$

da cui $0 < -\lg x < \varepsilon \frac{1}{\sqrt{x}}$

Dunque $\int_0^1 -\lg x$ esiste finito perché $-\lg x$ è maggiorata da una funzione integrabile $\frac{1}{\sqrt{x}} \geq 0$.

$$\int_0^1 \ln \frac{1}{x}, \quad \int_0^{+\infty} \frac{dx}{\sqrt{x}+x^3}, \quad \int_{-\infty}^{+\infty} \frac{1}{1+x^4}$$

$$\int_1^{+\infty} \frac{1}{x^3 \ln x}$$