Note Title 12/8/2021

$$\lim_{x\to 0} \frac{\cos\sqrt{x} - e^{\frac{x}{2}}}{\sin 2x + \log(1-x) + \operatorname{tg}(-x)}$$

$$\cos\sqrt{x} - 1 + 1 - e^{x/2}$$

$$\simeq \frac{(\sqrt{x})^2}{2} \qquad \frac{x}{2}$$

$$\cos\sqrt{x} = 1 - \frac{y^2}{2!} + \frac{y^4}{4!} + \cdots \Big|_{y=\sqrt{x}}$$

$$= 1 - \frac{x}{2} + \frac{x^2}{4!} \cdots$$

$$\underbrace{\sin 2x}_{\sim 2x} + \underbrace{\log(1-x)}_{\sim(-x)} + \underbrace{\operatorname{tg}(-x)}_{\sim -x}$$

occorre una tutta la serie di Taylor di $\cos y$, e^y, $\sin y$, $\log(1+y)$ e $\operatorname{tg} y$ e ottenere il grado minimo dei termini che sopravvivono.

$$f(x) - \sum_{0}^{n} \frac{f^{(k)}(x_0)}{k!}(x-x_0)^k = o\left(x-x_0\right)^n \qquad \text{Peano}$$

$$f(x) - \underbrace{\quad\quad}_{P_n(x)} = \boxed{\frac{f^{(n+1)}(\xi)}{(n+1)!}(x-x_0)^{n+1}} \qquad \text{Lagrange}$$

ξ interno a x_0, x

Il resto di Peano è utile perché ci permette di poter scegliere la distanza fra x_0 e x

Il resto di Lagrange è utile quando x_0 e x sono "fissi"

$$R = \sup\left\{ |x - x_0| : \sum_0^\infty a_n (x-x_0)^n \text{ converge} \right\}$$

$$\forall y : \underbrace{|y - x_0|}_{< \sup} < R \qquad \sum_0^\infty |a_n (y-x_0)^n| \text{ converge}$$

$$\Rightarrow \exists\, |\bar{x} - x_0| > |y - x_0|$$

$< \sup$

$\Rightarrow$ NON può essere maggiorante

$\boxed{\text{la serie converge}}$

$$|a_n (y-x_0)^n| = |a_n| \left| \frac{y-x_0}{\bar{x}-x_0} \right|^n |\bar{x}-x_0|^n$$

$|a_n||\bar{x}-x_0|^n \longrightarrow 0$ perché vale la CN convergenza

$\Rightarrow |a_n||\bar{x}-x_0|^n$ è limitata, cioè

$\exists M : |a_n||\bar{x}-x_0|^n \leq M$

da cui $\underbrace{\leq}_{0} \underbrace{|a_n (y-x_0)^n|} \leq M \underbrace{\left| \frac{y-x_0}{\bar{x}-x_0} \right|^n}_{<1}$

$<1 \Rightarrow$ termine generale di una serie geom. convergente

$\int_a^c f(t)\,dt$ esiste

$\int_c^b f(t)\,dt$ esiste $\Longrightarrow$ $\int_a^b f(t)\,dt = \int_a^c + \int_c^b$

CNS Dato ε so sceglie $\frac{\varepsilon}{2}$ e, in corrisp., $\exists\ \Pi$ in $[a,c]$ e Π' in $[c,b]$ tale che $|\Sigma_\Pi - \sigma_\Pi| < \frac{\varepsilon}{2}$ e $|\Sigma_{\Pi'} - \sigma_{\Pi'}| < \frac{\varepsilon}{2}$

$\Pi'' = \Pi \cup \Pi'$ è una partizione di $[a,b]$ che contiene tutti i punti di Π e Π' =

$$|\Sigma_{\Pi''} - \sigma_{\Pi''}| = |\underbrace{\Sigma_\Pi + \Sigma_{\Pi'}}_{\Sigma_{\Pi''}} - \underbrace{\sigma_\Pi - \sigma_{\Pi'}}_{-\sigma_{\Pi''}}| =$$

$$= \left| \Sigma_{\pi} - \sigma_{\pi} + \Sigma_{\pi'} - \sigma_{\pi'} \right| \le \underbrace{\left| \Sigma_{\pi} - \sigma_{\pi} \right|}_{< \frac{\varepsilon}{2}} + \underbrace{\left| \Sigma_{\pi'} - \sigma_{\pi'} \right|}_{< \frac{\varepsilon}{2}} <$$

$$< \frac{\varepsilon}{2} + \frac{\varepsilon}{2} = \varepsilon$$

$\uparrow$ CS d'integrab.

$\forall \varepsilon \ \exists \pi'' : \Sigma_{\pi''} - \sigma_{\pi''} < \varepsilon \Rightarrow f$ è integrabile su $[a,b]$

$$\sigma_{\pi''} \le \int_a^b \le \Sigma_{\pi''}$$

$$\sigma_{\pi''} = \sigma_{\pi} + \sigma_{\pi'} \qquad \Sigma_{\pi''} = \Sigma_{\pi} + \Sigma_{\pi'}$$

$$\sigma_{\pi} \le \int_a^c \le \Sigma_{\pi}$$

$$\sigma_{\pi'} \le \int_c^b \le \Sigma_{\pi'}$$

$$\le \int_a^c + \int_c^b \le$$

$$\left| \int_a^b f - \int_a^c f - \int_c^b f \right| \le \Sigma_{\pi''} - \sigma_{\pi''} < \varepsilon$$

	x_1	x_2	x_3		
I	1	1	1	3	
II	2	1	3	2 ✓	II − 2I
III	1	−1	3	1 ✓	
II	0	−1	1	−4	
III	0	−2	2	−2 ✓	III − I
III	0	1	−1	1 ✓	III/(−4)
					III + II
	0	0	0	$-3 \neq 0$	

FINE DEL LAVORO!

		x_1	x_2	x_3		
→	✓ I	(1)	1	1	2	
→	✓ II	2	−1	5	1	
→	✓ III	1	1	1	2	
↦	✓ II	0	−3	3	−3	II − 2I
	II	0	(1)	−1	1	II / (−3)
	III	0	0	0	0	III − I
	I	(1)	0	2	1	I − II

x_1	x_2	=	x_3
(1)	0	1	−2
0	1	1	1

$x_1 = 1 - 2x_3$

$x_2 = 1 + x_3$

II ⇄ I 2×

$$\begin{pmatrix} 1 - 2x_3 \\ 1 + x_3 \\ 0 + x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + x_3 \begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$$

$$\int \frac{x^2-2}{x^2(x+1)}\,dx$$

$$\frac{x^2-2}{x^2(x+1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} = \frac{A x(x+1) + B(x+1) + C x^2}{x^2(x+1)}$$

$$ax^2+bx+c$$

$$\frac{A}{x} + \frac{B}{x^2} + \frac{C}{x+1} \equiv 0$$

$$\lim_{x\to -1} \frac{A(x+1)}{x} + \frac{B(x+1)}{x^2} + C \equiv 0$$

$$C = 0$$

$\forall \varepsilon > 0 \quad \exists \nu : \forall n, m > \nu \qquad a_m - \varepsilon < a_m < a_m + \varepsilon$

$\varepsilon = \frac{1}{k}$

$\nu_k > \nu_{k-1} \quad \forall k$

$a_{\nu_{k+1}} - \frac{1}{k} < a_m < a_{\nu_{k+1}} + \frac{1}{k} \quad \longleftarrow \quad \forall m > \nu_k$

$$I_k = \left[a_{(\nu_{k+1})} - \frac{1}{k} , \; a_{(\nu_{k+1})} + \frac{1}{k} \right]$$

$I_{k+1} \subseteq I_k$ jestli

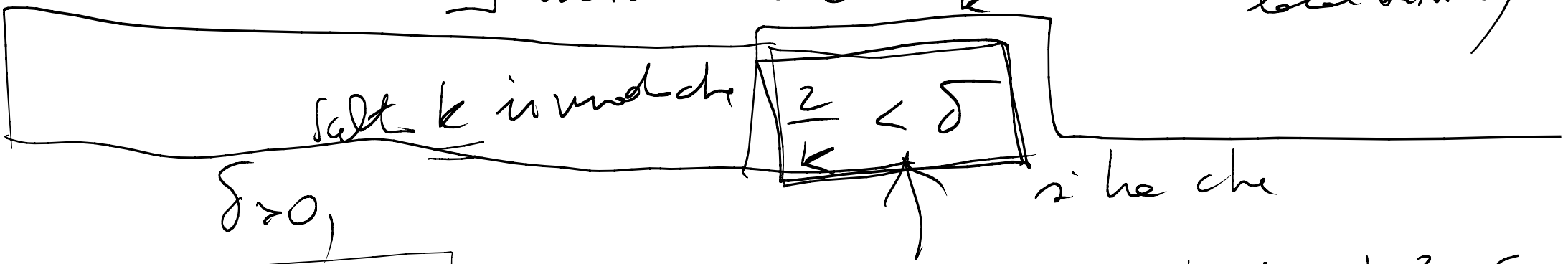
$a_{(\nu_k+1)} - \frac{1}{k} \leq a_{\nu_{k+1}} \leq a_{(\nu_{k+1})} + \frac{1}{k}$

$\nu_k \Rightarrow \in I_k$

$$a_{\nu_{k+1}} - \frac{1}{k} \leq a_{\nu_{k+1}} - \frac{1}{k+1} \leq a_{\nu_{k+1}} + \frac{1}{k+1} \leq a_{(\nu_k+1)} + \frac{1}{k}$$

$$|I_k| = \frac{2}{k} \xrightarrow[k\to\infty]{} 0 \qquad I_k \supseteq I_{k+1} \ \forall k$$

$\exists$ unico $L \in I_k \ \forall k$ (principio delle scatole cinesi)



$\delta > 0$,

$\forall\, y \in I_k$ (poiché $L \in I_k \ \forall k$) $\quad |y - L| \leq |I_k| = \frac{2}{k} < \delta$

$\boxed{a_n \in I_k}$ $\forall n > \nu_k$ $\quad y = a_k$

$|a_n - L| < \delta \quad \forall n > \nu_k$