Note Title 12/7/2021

① $\int \frac{x-1}{x^2(x+1)}$

$$\frac{x-1}{x^2(x+1)} = \frac{A}{x+1} + \frac{B}{x} + \frac{C}{x^2} = \frac{\quad}{\quad}$$

$$\int \frac{1}{1+x^2}\,dx = \operatorname{arctg} x$$

$$\int \frac{1}{(1+x^2)^2}\,dx$$

$$\frac{1}{(1+x^2)^2} = \frac{1+x^2}{(1+x^2)^2} - \frac{x^2}{(1+x^2)^2} =$$

$$\int \frac{x^2}{(1+x^2)^2}\,dx = \int \frac{x}{(1+x^2)^2} \cdot x\,dx = \frac{1}{2}\int \frac{2x}{(1+x^2)^2} \cdot x =$$

$$\int \frac{2x}{(1+x^2)^2}\,dx \overset{1+x^2=y}{=\!=} \int \frac{1}{y^2}\,dy = -\frac{1}{y} = -\frac{1}{1+x^2}$$

$2x\,dx = dy$

$$= \frac{1}{2}\left[-\frac{x}{1+x^2} + \int \frac{1}{1+x^2}\right] = -\frac{1}{2}\frac{x}{1+x^2} + \frac{1}{2}\arctg x$$

ERRATA-CORRIGE

$$\int \frac{1}{(1+x^2)^n}\,dx = I_n$$

$$I_n = \int \frac{1}{(1+x^2)^n} = \int \frac{1+x^2}{(1+x^2)^n} - \int \frac{x^2}{(1+x^2)^n} = \int \frac{1}{(1+x^2)^{n-1}} - \frac{1}{2}\int \frac{2x}{(1+x^2)^n}\,x\,dx =$$

$$= I_{n-1} - \frac{1}{2}\left[\frac{1}{1-n}\,\frac{x}{(1+x^2)^{n-1}} - \int \frac{1}{1-n}\,\frac{1}{(1+x^2)^{n-1}}\,dx\right]$$

$$= I_{n-1} - \frac{1}{2(1-n)}\,\frac{x}{(1+x^2)^{n-1}} + \frac{1}{2}\,\frac{1}{1-n}\,I_{n-1}$$

$$\int y^{-n} = \frac{1}{-n+1}\,y^{-n+1}$$

$$\int \frac{2x}{(1+x^2)^n}\,dx = \frac{1}{1-n}\,\frac{1}{(1+x^2)^{n-1}}$$

ERRATA - CORRIGE

$I_1 = \operatorname{arctg} x$

$$\int \frac{1}{x^2+2}\,dx = \frac{1}{2}\int \frac{1}{\frac{x^2}{2}+1}\,dx = \frac{1}{2}\int \frac{1}{\left(\frac{x}{\sqrt{2}}\right)^2+1}\,dx \quad \left|\ \frac{x}{\sqrt{2}} = y\right.$$

$$x = \sqrt{2}\,y$$

$$dx = \sqrt{2}\,dy$$

$$= \frac{1}{2}\int \frac{1}{y^2+1}\sqrt{2}\,dy = \frac{1}{\sqrt{2}}\operatorname{arctg} y = \frac{1}{\sqrt{2}}\operatorname{arctg}\frac{x}{\sqrt{2}}$$

$$\int \frac{1}{x^2+2x+4}\,dx =$$

$$x^2+2x+4 =$$

$$= x^2+2x+1+3 =$$

$$= (x+1)^2+3$$

$$= \int \frac{1}{(x+1)^2+3}\,dx = \frac{1}{3}\int \frac{1}{\left(\frac{x+1}{\sqrt{3}}\right)^2+1}\,dx \quad \left|\ y = \frac{x}{\sqrt{3}} + \frac{1}{\sqrt{3}}\right.$$

$$dy = \frac{1}{\sqrt{3}}\,dx \qquad dx = \sqrt{3}\,dy$$

$$= \frac{1}{3} \int \frac{1}{y^2+1} \sqrt{3}\, dy = \frac{1}{\sqrt{3}} \operatorname{arctg}\left(\frac{x+1}{\sqrt{3}}\right)$$

$\exists x_0 \in \mathbb{C}$ $\underbrace{a_n x_0^n + a_{n-1} x_0^{n-1} + \dots + a_1 x_0 + a_0}_{p(x)} = 0$ $\Leftrightarrow$

$a_i \in \mathbb{R}$ $\forall i$

$$a_n x_0^n + \dots + a_1 x_0 + a_0 = 0 \Rightarrow$$

$$\overline{a_n x_0^n + \dots + a_1 x_0 + a_0} = 0$$

$$\overline{a_n x_0^n} + \dots + \overline{a_1 x_0} + \overline{a_0} = 0$$

$$\overline{a_n}\left(\overline{x_0}\right)^n + \overline{a_{n-1}}\left(\overline{x_0}\right)^{n-1} + \dots + \overline{a_1}\,\overline{x_0} + \overline{a_0} = 0$$

$$a_n \overline{x_0}^n + a_{n-1} \overline{x_0}^{n-1} + \dots + a_1 \overline{x_0} + a_0 = 0$$

Ne segue che anche $\overline{x_0}$ è uno zero del polinomio originale $a_n x^n + \dots + a_1 x + a_0$

Se $p(x)$ si annulla in $x_0 \in \mathbb{C}$ e, di conseguenza, in $\overline{x_0}$, per il Th. Ruffini è divisibile per $(x - x_0)$ e per $(x - \overline{x_0})$, e quindi per il loro prodotto.

$$(x - x_0)(x - \overline{x_0}) = x^2 - \underbrace{(x_0 + \overline{x_0})}_{2 \operatorname{Re} x_0} x + \underbrace{x_0 \overline{x_0}}_{|x_0|^2}$$

$$\int \frac{x-1}{x^2+2x+2}\,dx$$

$$(x+1)^2+1$$

$$\frac{1}{2}\int \frac{2x+2}{x^2+2x+2} - \frac{2}{x^2+2x+2}$$

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)|$$

$f(x)$ → y

$y=f(x)$

$dy = f'(x)\,dx$

$$\int \frac{x^2+1}{x(x^2+2x+2)}$$

$(x^2+2x+2)'$

$$\frac{x^2+1}{x(x^2+2x+2)} = \frac{A}{x} + \frac{B(2x+2)}{x^2+2x+2} + \frac{C}{x^2+2x+2}$$

$$A \ln|x| + B \ln|x^2+2x+2|$$

$$\int \frac{e^t}{e^t+1}\,dt$$

f rapporto di polinomi di 2 variabili

$$\frac{p(x)}{q(x)} \quad , \quad x = e^t$$

$$\frac{y}{y+1}\Big|_{y=e^t}$$

$$f(e^t)$$

$$\int \frac{e^t}{e^t+1}\,dt = \int \frac{x}{x+1}\cdot\frac{1}{x}\,dx = \lg|x+1|\Big|_{x=e^t} \qquad x>0$$

$e^t = x$

$x>0$

$t = \lg x$

$dt = \frac{1}{x}\,dx$

$$\lg(1+e^t)$$