Note Title 12/3/2021

f continua su $[a,b]$

$$\left[\int_a^x f(t)\,dt\right]' = f(x) \quad \forall x \in]a,b[\qquad \Big| \quad F(x) = \int_a^x f(t)\,dt$$

$\Downarrow$

$$F'(x) \equiv f(x)$$

F è una primitiva di f su $]a,b[$.

$$\big(F + \underbrace{\text{cost}}\big)' \equiv F' \equiv f$$

2) $\boxed{G(x) - G(x_0) = \int_{x_0}^{x} f(t)\,dt}$ $\Rightarrow$ $G' \equiv f$

Barrow

Dim. $\forall$ G primitiva di f ho

$$\left[G(x) - \underbrace{\int_a^x f(t)\,dt}_{F(x)} \right]' = \underbrace{G'(x)}_{\| \; f(x)} - f(x) \equiv 0$$

Poiché siamo su un intervallo $\Rightarrow$ $G(x) - F(x)$ è costante, diciamo $G(x) - F(x) = G(a) - F(a)$, cioè

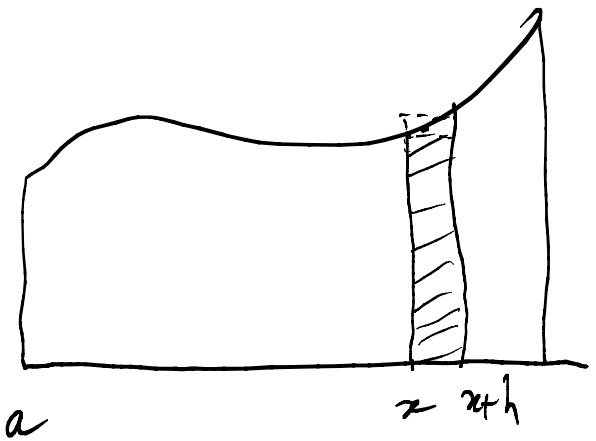
$$G(x) - \int_a^x f(t)\,dt = G(a) - \int_a^a f(t)\,dt$$

$$\int_a^a f(t)\,dt = 0$$

$$G(x) - G(a) = \int_a^x f(t)\,dt$$

$$\int_0^1 x^2\,dx \qquad \left(\frac{x^3}{3}\right)' \equiv x^2 \Rightarrow \int_0^1 x^2\,dx = \frac{x^3}{3}\Big|_0^1 \equiv$$

$$\equiv \frac{1^3}{3} - \frac{0^3}{3} = \frac{1}{3}$$



$$F(x) = \int f(x)\,dx \quad \text{se} \quad F'(x) \equiv f(x)$$

$$\left(\int f(x)\,dx\right)' \equiv f(x)$$

$\int f$ è una primitiva (qualsiasi) di f

$$\int \frac{1}{x}\,dx$$

$$\left[\lg|x|\right]' = \frac{1}{|x|}\cdot\frac{x}{|x|} = \frac{1}{x}$$

$$\frac{x}{|x|} = \begin{cases} 1 & x>0 \\ -1 & x<0 \end{cases}$$

$$\left(\lg|x|\right)' \equiv \frac{1}{x}$$

Tutte le primitive saranno del tipo $\frac{1}{x} + \begin{cases} k_1 & x>0 \\ k_2 & x<0 \end{cases}$

$$\int f+g = \int f + \int g$$

$$\int cf = c\int f$$

$$\neq' \equiv cf \qquad cf$$

$$\left[F(x)G(x)\right]' = F'(x)G(x) + F(x)G'(x)$$

$F'(x) \equiv f(x)$

$G'(x) = g(x)$

$$\left[f(x)g(x)\right]' = f'(x)g(x) + f(x)g'(x)$$

$$f(x)g(x) = \int f'(x)g(x) + \int f(x)g'(x)$$

INTEGRAZIONE PER PARTI

$$\int f'(x)\,g(x)\,dx = f(x)\,g(x) - \int f(x)\,g'(x)$$

$$\int x \sin x\,dx = x(-\cos x) - \int (-\cos x)\cdot(x)'\,dx =$$

$g(x)$ $f(x)$

$\int \sin x = -\cos x$

$$= -x\cos x + \int \cos x\,dx = -x\cos x + \sin x$$

$$\int x \lg x = \frac{x^2}{2}\lg x - \int \frac{x^2}{2}\cdot\frac{1}{x} = \boxed{\frac{x^2}{2}\lg x - \frac{x^2}{4}}$$

f' g f

$\frac{1}{2}\int x = \frac{1}{2}\cdot\frac{x^2}{2}$

$$\int \operatorname{tg} x = \int \underset{f'}{1} \underset{g}{\operatorname{tg} x} = x \operatorname{tg} x - \int \cancel{x} \cdot \frac{1}{\cancel{x}} = x \operatorname{tg} x - x$$

$$\int \operatorname{arctg} x = \int 1 \cdot \operatorname{arctg} x = x \operatorname{arctg} x - \underbrace{\int x \cdot \frac{1}{1+x^2} dx}_{?}$$

$$\left[f(g(x)) \right]' = f'(g(x)) \, g'(x)$$

$$\underbrace{f(g(x))}_{\int \frac{\operatorname{tg} x}{x}} = \underbrace{\int f'(g(x)) \, g'(x) \, dx}_{(\operatorname{tg} x)' = \frac{1}{x}}$$

$$\underline{f(y) = \int f'(y) \, dy}$$

$$\int \underbrace{\ln x}_{g(x)} \cdot \underbrace{\frac{1}{x}}_{g'(x)} dx =$$

$$= \frac{\ln^2 x}{2}$$

$f(g(x)) = g(x)$

$f(t) = t$

$$\boxed{F(t) = \frac{t^2}{2}}$$

$y = g(x)$

$$\int \underbrace{\cos}_{f'}\left(\overbrace{x^2}^{g(x)}\right) \overbrace{2x}^{g'(x)} dx =$$

$\cos t = f'(t)$

$\leftarrow g(x) = x^2$

$g'(x) = 2x$

$$= \sin(x^2)$$

$$\int \frac{e^t}{1+e^t}\,dt \underset{e^t = x}{=} \int \frac{\cancel{x}}{1+x}\cdot\frac{1}{\cancel{x}}\,dx \qquad dt = \frac{dt}{dx}\,dx$$

$$\boxed{t = \lg x}$$

$$f(x) = \frac{x}{1+x} \qquad dt = (\lg x)'\,dx$$

$$x = e^t \qquad dt = \frac{1}{x}\,dx$$

$x \neq 0$

$$\int \frac{1}{1+x}\,dx = \lg|1+x| = \lg|1+e^t| = \lg(1+e^t)$$

$$\int \frac{f'(x)}{f(x)}\,dx \underset{y=f(x)}{=} \int \frac{1}{y}\,dy =$$

$$dy = f'(x)\,dx$$

$$= \lg|y|\Big|_{y=f(x)} = \lg|f(x)|$$

$$\int \frac{1}{x(x-1)}\,dx$$

$$\int \frac{1}{x}\,dx = \ln|x|$$

$$\int \frac{1}{x-1}\,dx = \ln|x-1|$$

$$\frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1} =$$

$$= \frac{A(x-1) + Bx}{x(x-1)} =$$

$$= \frac{(A+B)x - A}{x(x-1)}$$

$$\rightarrow \quad (A+B)x - A \equiv 1 \Rightarrow \begin{cases} -A = 1 \\ (A+B) = 0 \end{cases}$$

$A = -1 \quad B = 1$

$$\frac{-1}{x} + \frac{1}{x-1} = \frac{1}{x(x-1)}$$

$$\int \frac{1}{x(x-1)}\,dx = \int -\frac{1}{x} + \int \frac{1}{x-1} = -\lg|x| + \lg|x-1|$$

$$\lg\left|\frac{x-1}{x}\right|$$

VIA RAPIDA

$$\frac{A}{x} + \frac{B}{x-1} = \frac{1}{x(x-1)} \qquad (\star)$$

Moltiplicando per x

$$A + \frac{Bx}{x-1} = \frac{1}{x-1}$$

facendo $x \to 0$

$$A + 0 = -1 \Rightarrow A = -1$$

Multiplicando (*) por $x-1$ se ha

$$\frac{A(x-1)}{x} + B = \frac{1}{x}$$

haciendo tender $x \to 1$ $\boxed{B=1}$

$$\frac{x^3+1}{(x+1)(x-1)}$$

x^2-1

x^3+1 | x^2-1

x^3-x | x

$= x+1$

$x^3+1 = x(x^2-1) + (x+1)$

dividendo = divisor · cociente + resto

$$\frac{x^3+1}{x^2-1} = \frac{x(x^2-1)}{x^2-1} + \frac{x+1}{x^2-1}$$

grado del numeratore è < grado del denom.

$$\int x^n = \frac{x^{n+1}}{n+1}$$

$$\frac{1}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1} = \frac{\dots}{x^2(x-1)}$$

Molt. per x^2 e facendo $x \to 0 \Rightarrow -1 = B$

Molt. per $(x-1)$ e facendo $x \to 1 \Rightarrow 1 = C$

$$\int x^{-n} = \frac{x^{-n+1}}{-n+1}$$