Note Title 11/24/2021

$$\sum_{1}^{\infty} \frac{1}{n+\sqrt{n}}$$

CN è verificata

$$\frac{\frac{1}{n+\sqrt{n}}}{\frac{1}{n}} = \frac{n}{n+\sqrt{n}} \longrightarrow 1$$

$$\frac{1}{n+\sqrt{n}} = O\left(\frac{1}{n}\right) \iff \text{stesso comportamento}$$

$$\sum_{2}^{\infty} \frac{1}{\lg(n!)}$$

$$1 \cdot 2 \cdot 3 \cdot \ldots \cdot (n-1)\, n = n!$$

$$n \cdot n \cdot n \cdot \ldots \cdot n \cdot n = n^n$$

$$n! \leq n^n$$

$$0 < \lg n! \leq n \lg n$$

$$\frac{1}{\lg n!} \geq \frac{1}{n \lg n}$$

→ term. generale di una serie divergente

DIVERGE

$$\sum \left(\frac{e^{\frac{1}{n}} - 1}{\frac{1}{n}} - 1 \right)$$

il term. gen. $\not\to 0$

$$\| $$

$$\frac{e^{\frac{1}{n}} - 1 - \frac{1}{n}}{\frac{1}{n}} \sim \frac{1}{2n}$$

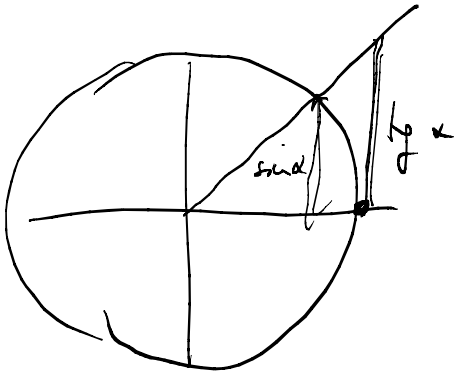
$$\frac{e^t - 1 - t}{t}$$

$$e^t = 1 + t + \frac{t^2}{2} + o(t^2)$$

$$\frac{\frac{t^2}{2} + o(t^2)}{t} = \frac{t}{2} + \frac{o(t^2)}{t} \to 0$$

$$\frac{o(t^2)}{t \cdot t} \cdot t$$

$$\sum \left(\sin \frac{1}{n} - \operatorname{tg} \frac{1}{n} \right)$$



$$\frac{\sin \frac{1}{n}}{\sim \frac{1}{n}} \cdot \frac{\left(1 - \frac{1}{\cos \frac{1}{n}} \right)}{\sim \frac{1}{2} \frac{1}{n^2}} =$$

$$= \left(\sin \frac{1}{n} \cdot \frac{\cos \frac{1}{n} - 1}{\cos \frac{1}{n}} \right)$$

$$\cos \frac{1}{n} - 1 \sim \frac{1}{n^2}$$

$$\sum \left[\left(1 + \frac{1}{n} \right)^n - e \right]$$

$$[\,] = e^{n \ln \left(1 + \frac{1}{n} \right)} - e =$$

$$= e \left[e^{n \ln \left(1 + \frac{1}{n} \right) - 1} - 1 \right] = (*)$$

$$n \log\left(1+\frac{1}{n}\right) - 1 = \frac{\log\left(1+\frac{1}{n}\right)}{\frac{1}{n}} - 1 \qquad n \to \infty,\ \frac{1}{n} \to 0$$

$$\frac{\log(1+t)}{t} \to 1 \qquad t = \frac{1}{n}$$

$$(*)\quad e^{\frac{e^{n\log\left(1+\frac{1}{n}\right)-1} - 1}{n\log\left(1+\frac{1}{n}\right)-1}} \cdot \left(\frac{\log\left(1+\frac{1}{n}\right)}{\frac{1}{n}} - 1\right)$$

$$\frac{\log(1+t)}{t} - 1 = \frac{\log(1+t)-t}{t} = \frac{-\frac{t^2}{2}+o(t^2)}{t} \sim \frac{t}{2}$$

$$t - \frac{t^2}{2}$$

$$\sin\frac{1}{n}\left(\frac{\cos\frac{1}{n}-1}{\cos\frac{1}{n}}\right)$$

tende ad un lim
finito e $\neq 0$
$= a_n = O\left(\frac{1}{n^3}\right)$

$$= \frac{\sin\frac{1}{n}}{\frac{1}{n}} \cdot \frac{\cos\left(\frac{1}{n}\right)-1}{\frac{1}{n^2}} \cdot \frac{1}{\cos\frac{1}{n}} \cdot \frac{1}{\frac{1}{n^3}}$$

$\to 1$ $\quad \to -\frac{1}{2}$ $\quad \to 1$

SERIE A SEGNI ALTERNI

$\sum_{1}^{\infty} (-1)^n a_n$ con

1) $a_n \geq 0$
2) $\lim_n a_n = 0$
3) $a_n \geq a_{n+1} \quad \forall n \in \mathbb{N}$

TS. $\sum_{1}^{\infty} (-1)^n a_n$ è convergente.

(CRITERIO DI LEIBNITZ)

Leibniz

Esempio

$\sum (-1)^n \frac{1}{n}$ NON CONVERGE ASSOLUTAM. (diverge $\frac{1}{n}$)

CONVERGE perché 1) $\frac{1}{n} > 0$ 2) $\lim \frac{1}{n} = 0$ 3) $\frac{1}{n}$ è decrescente

$$-a_1 + a_2 - a_3 + a_4 - a_5 \dots$$

$$S_n = \sum_1^n (-1)^k a_k \quad \left(\begin{array}{l}\text{somma parziale di}\\ \text{indice } n\end{array}\right)$$

$$S_{2k+2} = S_{2k} - \underbrace{a_{2k+1} + a_{2k+2}}_{\leq 0} \leq S_{2k}$$

Le somme parziali di indice pari formano una successione decrescente

$$S_{2k+1} = S_{2k-1} + \underbrace{a_{2k} - a_{2k+1}}_{\geq 0} \geq S_{2k-1}$$

Quelle di indice dispari formano una successione crescente

$$S_{2k} \geq S_{2k} - a_{2k+1} = S_{2k+1}$$

$$S_1 \leq S_3 \leq S_5 \leq \dots \leq S_{2k+1} \leq S_{2k} \leq S_{2k-2} \leq$$

$$\dots \leq S_4 \leq \boxed{S_2} \text{ maggiorata}$$

La successione $k \to S_{2k+1}$ è crescente e limitata superiormente da S_2 $\Rightarrow$ converge a $S_{disp.}$

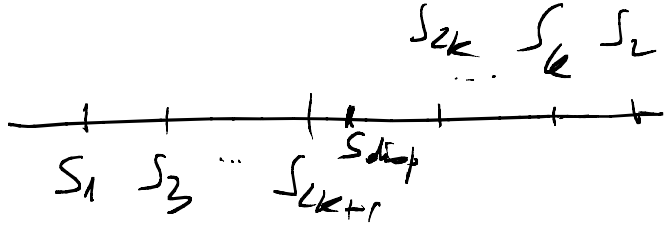
Analogamente

$$S_{2k} \geq S_{2k-1} \geq S_{2k-3} \geq \dots \geq \boxed{S_1} \text{ minorante}$$

Dunque $k \to S_{2k}$ converge (decrescendo) al proprio inf $= S_{peri}$

$S_{disp} \leq$

$S_1 \leq S_3 \leq S_5 \leq \dots \quad S_{2k}$



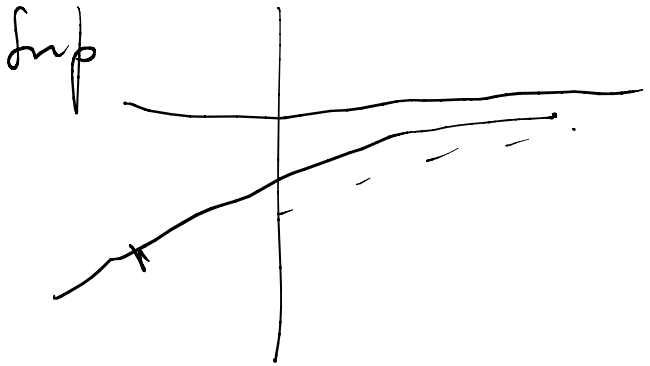
$S_{2k} \geq S_{2k+1}$

S_{2k} decresce

S_{2k+1} cresce

$S_{disp} = \lim S_{2k+1} =$

$= \sup_{k \geq h} S_{2k+1}$



$S_{disp} \leq S_{2k}$ $\forall k \Rightarrow S_{disp} \leq S_{peri}$

$$S_{2k+1} = S_{2k} - a_{2k+1}$$

$$\downarrow \qquad \downarrow \qquad \downarrow$$

$$S_{disp.} = S_{pari} \qquad 0$$

$$\downarrow$$

$$S_{2k+1} \leq S_{2k} \leq \dots S_{2h} \leq \dots \leq S_2$$

$$h < k$$