Note Title 11/23/2021

$\sum \frac{1}{n^p}$ $\quad p = 1$ serie armonica

$p \neq 1$ serie armonica generalizzata

$p \leq 1$ divergono

$p > 1$ convergono.

$\sum \frac{1}{n \log^p n}$

Criterio di Cauchy $\sum_{n=1}^{\infty} a_n$, $a_n \geq 0$, a_n decrescente,

converge o diverge se lo fa la serie $\sum_{k=1}^{\infty} 2^k a_{2^k}$

indice

$\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge (o diverge) a seconda che p

faccio $\sum_{k=1}^{\infty} 2^k \frac{1}{(2^k)^p} = \sum \frac{1}{2^{k(p-1)}} =$ $\boxed{p>1}$

$= \sum \frac{1}{\left[2^{(p-1)}\right]^k}$ serie geometrica di ragione 2^{p-1}

se $p>1$ $p-1>0 \Rightarrow 2^{p-1} > 1 \Rightarrow$

$\sum_{k=1}^{\infty} \left(\frac{1}{2^{p-1}}\right)^k$ converge.

Nel caso $p \leq 1$ $\quad \frac{1}{n^p} \geq \frac{1}{n}$

e dunque $\sum \frac{1}{n^p}$ diverge per confronto con

$\sum \frac{1}{n}$ (serie armonica)

$\sum_2^\infty \frac{1}{n \lg^p n}$ si comporta come

$$\frac{1}{n \lg n \lg\lg n} \quad \cdots \quad \frac{1}{n \lg n \lg\lg n}$$

$$\sum_{k=2}^{\infty} \cancel{2^k} \frac{1}{\cancel{2^k} (\lg 2^k)^p} = \sum_2^\infty \frac{1}{(k \lg 2)^p} = \frac{1}{(\lg 2)^p} \sum_{k=2}^{\infty} \frac{1}{k^p}$$

a_n decrescente

$a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + \dots + a_n$ → $n < 2^k$

$\wedge 1$ $\wedge 2$

$a_1 + \overline{a_2 + a_2} + \overline{a_4 + a_4 + a_4 + a_4} + \dots\dots + \overline{a_{2^k} + \dots a_{2^{k+1}-1}}$

$2a_2$ $4a_4$ $\wedge$

$a_{2^k} + a_{2^k} + \dots + a_{2^k}$

$2^k a_{2^k}$

$$\sum_{i=1}^{n} a_i \leq \sum_{j=1}^{k} 2^j a_{2^j}$$

Somma parziale della serie di partenza

somma parziale della serie $\sum 2^k a_{2^k}$ ←

se $\sum_{j=1}^{\infty} 2^j a_{2^j}$ converge, la successione delle

sue somme parziali è maggiorata da $\sum_{1}^{\infty} 2^k a_{2^k}$

da cui segue $\sum_{i=1}^{n} a_i \leq \sum_{1}^{\infty} 2^k a_{2^k}$

da cui la successione delle somme parziali di $\sum_{1}^{\infty} a_n$ è limitata e quindi $\sum_{1}^{\infty}$ converge

$$a_1 + a_2 + \underbrace{a_3 + a_4}_{\geq} + \underbrace{a_5 + a_6 + a_7 + a_8}_{\geq} + \cdots + a_n \quad \cdots$$

$\geq$ $\geq$ $\geq$ $\geq$

$$a_1 + a_2 + \underbrace{a_4 + a_4}_{2a_4} + \underbrace{a_8 + a_8 + a_8 + a_8}_{4a_8}$$

$\geq$

$\frac{1}{2} a_1$

$n > 2^k$

$$\sum_{1}^{n} a_i \geq \frac{1}{2}\left(a_1 + 2a_2 + 4a_4 + 8a_8 \cdots\right)$$

$$\sum_{1}^{n} a_i \geq \frac{1}{2} \sum_{j=1}^{k} 2^j a_{2^j}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^n}$$

$$\sqrt[n]{\frac{1}{n^n}} = \frac{1}{\sqrt[n]{n^n}} = \frac{1}{n} \longrightarrow 0$$

$$\frac{\frac{1}{(n+1)^{n+1}}}{\frac{1}{n^n}} = \frac{1}{(n+1)} \frac{\frac{1}{(n+1)^n}}{\frac{1}{n^n}} =$$

$$= \frac{1}{n+1} \frac{1}{\left(\frac{n+1}{n}\right)^n} = \frac{1}{n+1} \cdot \frac{1}{\left(1+\frac{1}{n}\right)^n} \longrightarrow 0$$

$\frac{1}{n+1} \to 0$

$\left(1+\frac{1}{n}\right)^n \to \frac{1}{e}$

$$n^n \geq n^2 \quad \forall\, n \geq 2$$

$$\frac{1}{n^n} \leq \frac{1}{n^2}$$

$$\sum_0^\infty \frac{x^n}{n!} \quad \left(= e^x \quad \forall x \in \mathbb{R} \right)$$

$$\frac{\frac{|x^{n+1}|}{(n+1)!}}{\frac{|x^n|}{n!}} = \frac{|x|}{n+1} \to 0$$

$$\sum \left(\sqrt{n+1} - \sqrt{n} \right)^3$$

$$\sqrt{n+1} - \sqrt{n} = \frac{n+1-n}{\sqrt{n+1} + \sqrt{n}} =$$

$$= \frac{1}{\sqrt{n+1} + \sqrt{n}} \longrightarrow 0$$

$$\frac{\left(\frac{1}{\sqrt{n+1} + \sqrt{n}}\right)^3}{\left(\frac{1}{\sqrt{n}}\right)^3} = \left(\frac{\sqrt{n}}{\sqrt{n+1} + \sqrt{n}}\right)^3$$

$$\frac{\sqrt{n}}{\sqrt{n}} \cdot \frac{1}{\sqrt{\frac{n+1}{n}} + 1}$$

$$\sqrt{\frac{n+1}{n}} \longrightarrow 1$$

$$\sum \left[\operatorname{arctg}(n+1) - \operatorname{arctg} n \right]$$

Lagrange

$$\operatorname{arctg}(n+1) - \operatorname{arctg} n = \underbrace{\left[(n+1) - n \right]}_{1} \frac{1}{1+\xi^2} \leq \qquad \xi \in]n, n+1[$$

$$\leq 1 \cdot \frac{1}{1+n^2} \leq \frac{1}{n^2} \qquad \xi > n$$