Note Title 11/17/2021

$$\lim_{x \to 0} \frac{x - \lg(1+x)}{x^2} =$$

$$= \lim_{0} \frac{\cancel{x} - \left(\cancel{x} - \frac{x^2}{2} + o(x^2)\right)}{x^2} =$$

$$= \lim_{0} \frac{\frac{x^2}{2} - o(x^2)}{x^2} = \frac{1}{2}$$

$$\underset{f(x)}{\lg(1+x)} = \underset{f(x_0)}{\lg 1} = f(0) \quad x_0 = 0$$

$$f'(x) = \frac{1}{1+x} \quad f'(0) = 1$$

$$f''(x) = -\frac{1}{(1+x)^2}$$

$$f''(0) = -1$$

$$\lg(1+x) = 0 + \underset{\frac{1}{1!} f'(0)(x)^1}{x} - \frac{1}{2}x^2 + o(x^2)$$

$$\lim_{x \to x_0} \frac{\overset{0 \leftarrow}{f(x)}}{\underset{\downarrow\, 0}{g(x)}} \underset{\text{Taylor}}{=} \lim_{x_0} \frac{\frac{1}{k!} f^{k}(x_0)(x-x_0)^k + o(x-x_0)^k}{\frac{1}{h!} g^{(h)}(x_0)(x-x_0)^h + o(x-x_0)^h}$$

di f e g

$k > h$

$$\frac{\left[\frac{1}{k!} f^{k}(x_0) + \overset{\to 0}{\frac{o(x-x_0)^k}{(x-x_0)^k}}\right](x-x_0)^k}{\left[\frac{1}{h!} g^{(h)}(x_0) + \underset{\downarrow\, 0}{\frac{o(x-x_0)^h}{(x-x_0)^h}}\right](x-x_0)^h} \qquad (x-x_0)^{k-h}$$

$\frac{1}{k!} f^{(k)}(x_0)$

$\xrightarrow{x \to x_0} \frac{1}{h!} g^{(h)}(x_0)$

$$\lim_{0^+} \frac{\sqrt{x} - \sin\sqrt{x}}{x} = \quad t = \sqrt{x}$$

$$t - \sin t$$

$$\boxed{\sin t} = 0 + t + \cancel{\frac{-\sin 0}{2!} t^2} + o(t^2)$$

$$= \lim_{t \to 0} \frac{t - \sin t}{t^2} = \lim_{t \to 0} \frac{\cancel{t} - \cancel{t} - o(t^2)}{t^2} = 0$$

$$t - \sin t = O(x^3)$$

$$\sin t = t - \frac{t^3}{3!} + \underline{o(t^3)}$$

$$6$$

TH. TAYLOR (con resto di LAGRANGE)

$f \in C^{n+1}]a,b[$

$\forall x, x_0 \in]a,b[\quad \exists \xi \in]x_0, x[$

$$f(x) - \sum_{0}^{n} \frac{1}{k!} f^{(k)}(x_0)(x-x_0)^k = \frac{1}{(n+1)!} f^{(n+1)}(\xi)(x-x_0)^{n+1}$$

$$\boxed{\frac{\overbrace{f(x) - \sum_{0}^{n} \frac{1}{k!} f^{(k)}(x_0)(x-x_0)^k}^{f(x)-P(x) \, - \, \overbrace{f(x_0)-P(x_0)}^{=0}}}{(x-x_0)^{n+1}}} \underset{\text{CAUCHY}}{=} \frac{f'(\xi_1) - \sum_{k=1}^{n} \frac{1}{(k-1)!} f^{(k)}(x_0)(\xi_1 - x_0)^{k-1}}{(n+1)(\xi_1 - x_0)^n} =$$

$(x-x_0)^{n+1} - \underbrace{(x_0-x_0)^{n+1}}_{=0}$

$\exists \xi_1$
interno ad $]x_0, x[$
tale che

$$\underset{\substack{\text{CAUCHY}\\ x_0,\ \xi_1}}{=} \frac{f''(\xi_2) - \sum_{2}^{n} \frac{1}{(k-2)!} f^{(k)}(x_0)(\xi_2 - x_0)^{k-2}}{(n+1)n(\xi_2 - x_0)^{n-1}} \underset{n \text{ volte}}{=}$$

$$= \frac{f^{(n)}(\xi_n) - f^{(n)}(x_0)}{(n+1)n(n-1)\cdots 2\cdot(\xi_n - x_0)} = \boxed{\frac{1}{(n+1)!}}\ \frac{f^{(n)}(\xi_n) - f^{n}(x_0)}{(\xi_n - x_0)}$$

cauchy

$$\underset{\substack{\text{CAUCHY}\\ [x_0,\ \xi_n]}}{=} \boxed{\frac{1}{(n+1)!}\ \frac{f^{(n+1)}(\xi)}{1}}$$

$$\frac{f(x) - P(x)}{(x - x_0)^{n+1}} = \frac{1}{(n+1)!} f^{(n+1)}(\xi)$$

COME CALCOLARE UN VALORE APPROSSIMATO DI e (con precisione arbitraria)

TAYLOR (LAGRANGE)

$$e = e^1 \qquad e^0 = 1$$

$f(x) = e^x$ $\quad \underline{x_0 = 0}$ $\quad \underline{x = 1}$

$f(0) = 1$

$$e^1 = \boxed{1 + 1 + \frac{1}{2!} + \frac{1}{3!} + \cdots +} + \cdots + \frac{1}{n!} + \boxed{\frac{1}{(n+1)!} \cdot e^{\xi}}$$

ξ è interno a $[0,1]$

$$\text{RESTO}_n \leq \frac{3}{(n+1)!}$$

$$f(x) = f(x_0) + f'(x_0)(x-x_0) + \frac{1}{2!} f''(x_0)(x-x_0)^2 + \cdots + \frac{1}{n!} f^{(n)}(x_0)(x-x_0)^n$$

$+ \cdots$