Note Title 11/16/2021

$$f(x) = \underbrace{\sum_{k=0}^{n} \frac{1}{k!} f^{(k)}(x_0)(x-x_0)^k}_{\text{polinomio di Taylor}} + \underbrace{o\left((x-x_0)^n\right)}$$

$$\frac{f(x) - \sum_{0}^{n} \cdots}{(x-x_0)^n} \longrightarrow 0$$

$$(x-x_0)^n \qquad n(x-x_0)^{n-1}$$

$$\boxed{n(n-1)}(x-x_0)^{n-2}$$

$$\sum_{k=0}^{n} \boxed{\frac{1}{k!} f^{(k)}(x_0)\,(x-x_0)^k} = P_n(x)$$

$$\left[\frac{1}{k!} f^{(k)}(x_0)\,(x-x_0)^k\right]' = f^{(k)}(x_0)\,\frac{1}{k!}\,\cancel{k}\,(x-x_0)^{k-1}$$

$$\cancel{k}(k-1)\cdots 3\cdot 2\cdot 1 = (k-1)!$$

$$\left(P_n(x)\right)' = \sum_{k=1}^{n} \frac{1}{(k-1)!} f^{(k)}(x_0)\,(x-x_0)^{k-1}$$

dopo n derivate:

$$\lim_{x\to x_0} \frac{f^{(n)}(x) - f^{(n)}(x_0)}{n!} = 0$$

$\exists \xi$ interno a $[a,b]$

$$\frac{f(b)-f(a)}{g(b)-g(a)} = \frac{f'(\xi)}{g'(\xi)}$$

$$\boxed{h(x) = \lambda f(x) + \mu g(x)}$$

$$h(a) = \lambda f(a) + \mu g(a)$$

$$h(b) = \lambda f(b) + \mu g(b)$$

$$\lambda\big(f(b)-f(a)\big) + \mu\big(g(b)-g(a)\big) = 0$$

(cambiato di segno)

$$\lambda = g(b)-g(a) \qquad \mu = -\big(f(b)-f(a)\big)$$

$$\exists\, \xi \in]a,b[\;:\; h'(\xi) = 0$$

$$\big[g(b)-g(a)\big] f'(\xi) - \big[f(b)-f(a)\big] g'(\xi) = 0$$

Dividendo per $g'(\xi)$ ($\neq 0$ per ipotesi) e per $g(b)-g(a)$ (che non può annullarsi perché Rolle implica $\exists \eta : g'(\eta)=0$

in contrasto all'ipotesi $f'(x) \neq 0$ on tutto $]a,b[$.

Formula di Taylor di ordine 1 $\quad f \in C^1$

$$f(x_0) + f'(x_0)(x - x_0) + o(x - x_0)$$

$$y = f(x_0) + f'(x_0)(x - x_0)$$