Note Title 11/12/2021

$$\lim_{x\to 0} (\cos x)^{\frac{1}{x}}$$

$$e^{\frac{1}{x}\log(\cos x)}$$

$$\lim_{x\to 0} \frac{\log(\cos x)}{x} = \lim_{x\to 0} \frac{\log(1+(\cos x-1))}{x} =$$

$$= \lim_{x\to 0} \frac{\log(1+(\cos x-1))}{\cos x - 1} \cdot \frac{\cos x - 1}{x\cdot x} \cdot x = 0$$

(t sopra $\log(1+(\cos x-1))$; $x \to 0$)

$t = \cos x - 1$

$$\frac{\log(1+t)}{t}$$

che non è definita nel punto 0

$\to 1$

$\to -\frac{1}{2}$

$x \to 0$

$$\lim_{x\to 0} (\cos x)^{\frac{1}{x}} = e^0 = 1$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

$$\lim_{x\to 0} \frac{\lg(1+x)}{x} = 1$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2}$$

$f(x), g(x)$ infinitesime in 0

$g(x) = O(f(x))$ se $\dfrac{g(x)}{f(x)}$ è limitata per $x \to 0$

(se sono infinitesimi in x_0, si richiede se $\dfrac{g(x)}{f(x)}$ sia limitata in un intorno di x_0

$\rightarrow \left| \frac{g(x)}{f(x)} \right| \leq k$ $\qquad |g(x)| \leq k|f(x)|$

$\rightarrow$ se qualche limite $\lim_{x \to x_0} \frac{g(x)}{f(x)} = \boxed{L \neq 0}$

$\bar{\varepsilon} < L$

$\Longrightarrow 0 < L - \bar{\varepsilon} < \frac{g(x)}{f(x)} < L + \bar{\varepsilon}$

$g = O(f)$ se $\frac{g(x)}{f(x)} \to L \neq 0$ in x_0

$g = o(f)$ se $\frac{g(x)}{f(x)} \to 0$

g è infinitesimo di ordine α rispetto ad f

$\longrightarrow$ se $g(x) = O(f^\alpha)$

$g(x) = \sin(x^2)$ $\quad f(x) = x$

$g = O(f^2)$ $\quad g$ è infinitesimo di ordine 2 rispetto a x

$\frac{\sin x}{x} \to 1$ per $x \to 0$

$\frac{\sin x^2}{x^2} \to 1$ per $x^2 \to t$

$\sin x = O(x)$

$$\frac{\ln(\cos x)}{x^2} \qquad \frac{\frac{-\sin x}{\cos x}}{2x} = -\frac{1}{2}\left(\frac{\sin x}{x}\right)\cdot\left(\frac{1}{\cos x}\right)$$

$\frac{\sin x}{x} \to 1$

$\frac{1}{\cos x} \to 1$

$$\lim_{x \to 0} \frac{\sin x}{x}$$

$$\lim_{x \to x_0} \frac{\sin x - \sin x_0}{x - x_0}$$

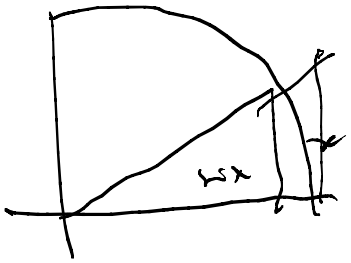
$$\sin x - \sin x_0 = 2 \sin \frac{x - x_0}{2} \cos \frac{x + x_0}{2}$$

$$\frac{\sin x - \sin x_0}{x - x_0} = \frac{\sin \frac{(x - x_0)}{2}}{\frac{x - x_0}{2}} \cdot \cos\left(\frac{x + x_0}{2}\right)$$

$\cos x_0$

$\frac{x - x_0}{2} = t$

Perché tende ad 1?



$$\lim_{x\to 0} \frac{\operatorname{arctg} x}{x^\alpha} = \qquad \alpha > 0$$

$$= \lim_{x\to 0} \frac{\frac{1}{1+x^2}}{\alpha x^{\alpha-1}} = \lim_{x\to 0} \frac{1}{\alpha(1+x^2)\, x^{\alpha-1}} \overset{\alpha=1}{=} 1$$

$$\lim_{x\to 0} \frac{\operatorname{arctg} x}{x} = 1$$

$g(x) = x^2 \lg x \qquad f(x) = x$

$$\lim_{x\to 0^+} \frac{x^2 \lg x}{x^\alpha}$$

$$\lim_{x\to 0} x^2 \lg x = \lim_{x\to 0} \frac{\lg x}{\frac{1}{x^2}} \overset{H}{=} \lim_{x\to 0} \frac{\frac{1}{x}}{-2\frac{1}{x^3}} = \lim_{x\to 1} -\frac{1}{2} \frac{x^3}{x} =$$

$$= 0$$

$\boxed{\alpha = 2}$ $\lim_{0^+} \frac{x^2 \log x}{x^2} = -\infty$

$\alpha > 2$ anche meglio $\log x \cdot \frac{1}{x^{\alpha-2}} = -\infty$

$\log x \to -\infty$, $\frac{1}{x^{\alpha-2}} \to +\infty$

$\alpha < 2$ $\lim_{x \to 0^+} \overbrace{x^{2-\alpha}}^{>0} \log x = \lim_{0^+} \frac{\log x}{\frac{1}{x^{2-\alpha}}} =$ $\frac{1}{x^{2-\alpha}} \to +\infty$, $= x^{\alpha-2}$

$\to \overset{H}{=} \lim_{0^+} \frac{\frac{1}{x}}{(\alpha-2)\, x^{\alpha-3}} = \lim_{0^+} \frac{1}{(\alpha-2)} \frac{\frac{1}{x}}{\frac{1}{x^{3-\alpha}}} \overset{\alpha}{\simeq} \lim_{0^+} \frac{x^{3-\alpha}}{x} =$

$\lim_{0^+} x^{2-\alpha} = 0$

$$\lim_{+\infty} \frac{x+\sqrt{x}}{3x+\cos x} = \lim_{\infty} \frac{x\left(1+\frac{\sqrt{x}}{x}\right)\to 1}{x\left(3+\frac{\cos x}{x}\right)\to 0} = \frac{1}{3}$$

RAPPORTI DI POLINOMI

$$\lim_{+\infty} \frac{x^m + \text{termini di grado più basso}}{x^m + \text{termini di grado più basso}}$$

$$\lim_{+\infty} \frac{x^3+3x-1}{x^2-2x-7} = \lim_{+\infty} \frac{x^3\left(1+\frac{3}{x^2}-\frac{1}{x^3}\right)\to 1}{x^2\left(1-\frac{2}{x}-\frac{7}{x^2}\right)\to 1} = +\infty$$

$x^3 \to +\infty$

$$\lim_{+\infty} x^2 - \sqrt{x^4+1} = \lim_{+\infty} x^2\left(1-\frac{\sqrt{x^4+1}}{x^2}\right)$$

$x^2 \to +\infty$

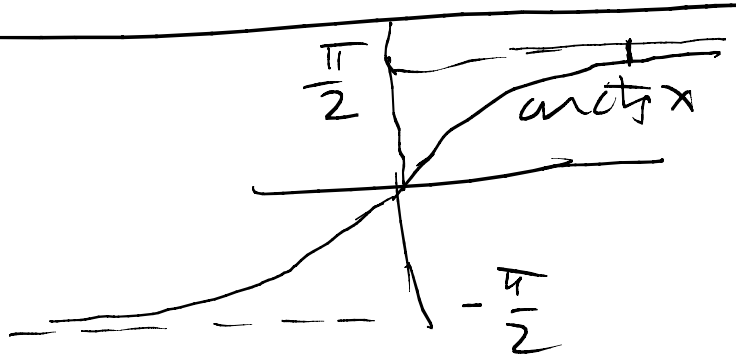
$\frac{\sqrt{x^4+1}}{x^2} \to 0$

$$\lim_{+\infty} \frac{x^2}{\sqrt{x^4+1}} = \lim_{+\infty} \sqrt{\frac{x^4}{x^4+1}} = 1$$

$$\frac{\left(x^2-\sqrt{x^4+1}\right)\left(x^2+\sqrt{x^4+1}\right)}{x^2+\sqrt{x^4+1}} = \frac{x^4-\left(\sqrt{x^4+1}\right)^2}{x^2+\sqrt{x^4+1}} =$$

$$= \frac{-1}{x^2+\sqrt{x^4+1}} \xrightarrow[x\to+\infty]{} 0$$

$$\lim_{x\to+\infty} \frac{\frac{\pi}{2}-\operatorname{arctg} x}{\frac{1}{x}} =$$



$$\overset{H}{=} \lim_{x\to+\infty} \frac{-\frac{1}{x^2+1}}{-\frac{1}{x^2}} = \lim_{x\to+\infty} \frac{x^2}{x^2+1} = 1$$

$$\lim_0 \frac{1}{x}\left[(1+x)^{\frac{1}{x}} - e\right] = \lim_0 \frac{e}{x}\left[e^{\frac{1}{x}\ln(1+x)-1} - 1\right] =$$

$$= e^{\frac{1}{x}\ln(1+x)}$$

$$= \lim_0 \frac{e}{x}\left[\frac{e^{\frac{1}{x}\ln(1+x)-1} - 1}{\frac{1}{x}\ln(1+x) - 1} \cdot \left(\frac{1}{x}\ln(1+x) - 1\right)\right] =$$

$$\frac{1}{x}\ln(1+x) - 1 \to 1$$

$$= e \lim_0 \frac{\frac{\ln(1+x)}{x} - 1}{x} = \left[\lim_0 \frac{\ln(1+x) - x}{x^2}\right] \overset{H}{=} \lim_0 \frac{\frac{1}{1+x} - 1}{2x}$$

$$= \lim_0 \frac{1 - 1 - x}{2x(x+1)} = \lim_0 -\frac{1}{2(1+x)} = \boxed{-\frac{e}{2}}$$

$$\uparrow$$

$$\lim_{0} \frac{a^{\alpha x} - 1}{a^{\beta x} - 1} \qquad \alpha > 0,\ \beta \neq 0 \qquad a \neq 1$$

$$a^{\alpha x} - 1 = e^{\alpha x \ln a} - 1$$

$$a^{\beta x} - 1 = e^{\beta x \ln a} - 1$$

$$\frac{e^{\alpha x \ln a} - 1}{\alpha x \ln a} \cdot \frac{\alpha x \ln a}{\beta x \ln a} \cdot \frac{\beta x \ln a}{e^{\beta x \ln a} - 1} \longrightarrow \frac{\alpha}{\beta}$$

$\alpha x \ln a = t$, $t \to 0$, $\to 1$

$\beta x \ln a = t$, $t \to 0$, $\to 1$

$$\overset{H}{=} \lim_{0} \frac{a^{\alpha x} (\ln a)\alpha}{e^{\beta x} (\ln a)\beta} = \lim_{0} a^{(\alpha - \beta)x} \cdot \frac{\alpha}{\beta} = \frac{\alpha}{\beta}$$

$a^{(\alpha-\beta)x} \to 1$

$$\frac{d}{dx} a^{\alpha x} = a^{\alpha x} \ln a \cdot \alpha$$

$$\lim_{0} (x+\cos x)^{\frac{1}{x}} = \lim_{0} e^{\frac{1}{x}\lg(x+\cos x)}$$

$$\lim_{x\to 0} \frac{\lg(x+\cos x)}{x} \overset{H}{=} \lim_{0} \frac{1}{x+\cos x}(1-\sin x) = 1$$

deriv. del numeratore

$$\lim_{+\infty} \frac{1}{\sin\frac{1}{x}} - x = \lim_{+\infty} x\left(\frac{1}{x\sin\frac{1}{x}} - 1\right) =$$

$$= \lim_{+\infty} \frac{\frac{1}{\frac{\sin\frac{1}{x}}{\frac{1}{x}}} - 1}{\frac{1}{x}} \underset{\frac{1}{x}=y}{=} \lim_{y\to 0} \frac{\frac{1}{\frac{\sin y}{y}} - 1}{y}$$

$$\frac{\frac{y}{\sin y} - 1}{y} = \frac{y - \sin y}{y \sin y} \longrightarrow 0$$

$y \to 0$

Hôpital

o elto.