Note Title 11/11/2021

$$\frac{f(x)}{g(x)} \to L \qquad \boxed{\frac{f'(x)}{g'(x)} \to L}$$

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f, g continue in $[x_0, x_0+\delta]$ $\quad f(x_0)=g(x_0)=0$

f, g derivabili $\quad g'(x)\neq 0$ in $]x_0, x_0+\delta[$

$$\lim_{x\to x_0^+} \frac{f(x)}{g(x)} = \lim_{x\to x_0^+} \frac{f'(x)}{g'(x)}$$

se esiste finito il $\lim_{x\to x_0^+} \dfrac{f'(x)}{g'(x)} = L$

$$\frac{f(x)}{g(x)} = \frac{f(x) - \overbrace{f(x_0)}^{=0 \text{ per Hp}}}{g(x) - g(x_0)} \underset{\text{CAUCHY}}{=} \frac{f'(\xi)}{g'(\xi)}$$

$$x_0 < \xi < x_0 + \delta$$

$\xi(x)$ è un punto che vale per il th. di Cauchy ed è INTERNO a $]x_0, x[$

$$y = \xi(x)$$

$$x_0 < \xi(x) < x \quad \leftarrow$$

$$\lim_{x \to x_0} \frac{f'(\xi(x))}{g'(\xi(x))} \underset{\boxed{y = \xi(x)}}{=} \lim_{y \to x_0} \frac{f'(y)}{g'(y)}$$

$$\boxed{\frac{f'(x)}{g'(x)}} \qquad \xi(x)$$

f continua in $[a,b]$, derivabile in $]a,b[$.

$$\exists \lim_{x \to a^+} f'(x) = L$$

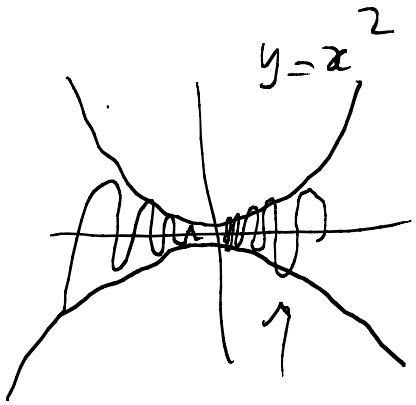
TS $\exists f'(a) = L$

Dim.

$$\lim_{x \to a^+} \frac{f(x) - f(a)}{x - a} \overset{H}{=} \lim_{x \to a^+} \frac{f'(x)}{1} = L$$

=

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & x \neq 0 \\ 0 & x = 0 \end{cases} \longleftarrow$$



$x \neq 0 \quad f'(x) = 2x \sin\frac{1}{x} + x^2 \cos\frac{1}{x}\left(-\frac{1}{x^2}\right) =$

$= \underbrace{2x \sin\frac{1}{x}}_{\to 0} - \cos\frac{1}{x}$ NON ESISTE

$\cos\frac{1}{x}$ oscilla tra -1 e 1

$$f'(0) = \lim_{x\to 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x\to 0} \frac{x^2 \sin\frac{1}{x} - 0}{x - 0} = \underbrace{\lim_{x\to 0} x \sin\frac{1}{x}}_{\to 0 \text{ limitato}} = 0$$

perché infinitesimo per limitato.

$$\lim_{x\to 0} \frac{\lg(1+x^2)}{\lg(1-x^2)}$$

dom f $\{x \neq 0\}$

BALLA SPAZIALE!

dom f $\boxed{1-x^2>0}$ $x^2<1$

$|x|<1$

$=]-1,1[\setminus \{0\}$ $x \in]-1,1[$

$$\lim_{x\to 0} \frac{\lg(1+x^2)}{\lg(1-x^2)} = \frac{\frac{\cancel{2x}}{1+x^2}}{-\frac{\cancel{2x}}{1-x^2}} \overset{x\neq 0}{=} -\boxed{\frac{1-x^2}{1+x^2}} = -1$$

$\lim_{x\to 0}$ $\lim_{x\to 0}$

$$\boxed{\frac{\lg(1+x^2)}{x^2}} \cdot \boxed{\frac{x^2}{-x^2}} \cdot \frac{-x^2}{\lg(1-x^2)}$$

$y=x^2$ $\to 1$ $\equiv -1$ per $x\neq 0$ $y=-x^2$ $\to 1$

$$\boxed{\lim_{x\to 0} \frac{\lg(1+x)}{x} = 1}$$

la funzione NON È DEF. in $x=0$

$$\lim_{x\to 0} \frac{e^x - 1}{\sin x} = \lim_{x\to 0} \underbrace{\frac{e^x - 1}{x}} \cdot \underbrace{\frac{x}{\sin x}}$$

$$\lim_{x\to 0} \frac{x - \sin x}{x^2 \sin x} \overset{H}{=} \lim_{x\to 0} \frac{1 - \cos x}{2x \sin x + x^2 \cos x} =$$

$$= \lim \frac{1}{\frac{2x \sin x + x^2 \cos x}{1 - \cos x}} = \lim \frac{1}{2x \cdot \frac{\sin x}{x} \cdot \frac{1}{1 - \cos x} + \frac{x \cdot \cos x}{1 - \cos x}} = \frac{1}{6}$$

$2x \to 2$, $\frac{\sin x}{x} \to 1$, $1 - \cos x \to 2$, $x \to 0$, $\cos x \to 1$, $\frac{x}{1 - \cos x} \to 2$

$$\lim_{x\to 1} \frac{\lg_x 2}{x-1}$$

$$\boxed{x>0}$$

$$\longrightarrow a^b = e^{b\lg a}$$

$$\underbrace{x^{\lg_x 2}} = 2$$

$$e^{\lg_x 2 \lg x} = 2$$

$$\Downarrow$$

$$\lg_x 2 \lg x = \lg 2$$

$$\boxed{\lg_x 2 = \frac{\lg 2}{\lg x}}$$

$x \to 1$

$x \to 1^+$

$$\lim_{x\to 1^+} \quad +\infty$$

$$\lim_{x\to 1^-} \quad +\infty$$

$$\lim_{x\to 1} (x-1)\lg_x 2$$

$$\frac{(x-1)\lg 2}{\lg x} \xrightarrow{x-1=t} \lg 2 \frac{t}{\lg(1+t)} \longrightarrow \lg 2$$

$$\lg x = \lg(1+(x-1))$$

$$\lg 2 \lim_{x\to 1} \frac{x-1}{\lg x} \overset{H}{=} \lg 2 \lim_{x\to 1} \frac{1}{\frac{1}{x}} = \lg 2 \lim_{x\to 1} x$$

$$\lim_{x\to 0^+} \sin x \lg x = \lim_{x\to 0^+} \underbrace{\frac{\sin x}{x}}_{\to 1} \cdot x \lg x$$

$$x \lg x = \frac{\lg x}{\frac{1}{x}}$$

$$\lim_{x\to 0^+} x \lg x \overset{H}{=} \lim_{x\to 0^+} \frac{1}{x} \cdot \frac{1}{-\frac{1}{x^2}} = \lim_{x\to 0^+} -x = 0$$

$$\lim_{x\to 0}\left(\frac{1}{\sin x}-\frac{1}{x}\right)=\lim_{x\to 0}\frac{x-\sin x}{x\sin x}=$$

$$=\lim_{x\to 0}\frac{x-\sin x}{x^2\sin x}\cdot x=0$$

($x\to 0$)

ACCIDENTI CHE GLI PARE (FINITO)

$$\lim_{x\to 0}(\cos x)^{\frac{1}{x}}$$

$$e^{\frac{1}{x}\lg(\cos x)}$$

$$\lim_{x\to 0}\frac{\lg(\cos x)}{x}\overset{H\wedge}{=}\lim_{x\to 0}\frac{1}{\cos x}(-\sin x)=0$$