Note Title 10/15/2021

$\frac{n^k}{A^n} \to 0$ $A > 1$

$\frac{A^n}{n!} \to 0$

$\frac{n!}{n^n} \to 0$

$0! = 1$ ←

$(n+1)! = (n+1)\,n!$ ←

$$\lim \frac{2^{n+1} + 3^{n+1}}{2^n + 3^n}$$

$$\frac{3^{n+1}\left(\left(\frac{2}{3}\right)^{n+1} + 1\right)}{3^n\left(\left(\frac{2}{3}\right)^{n} + 1\right)} \to 3$$

$\left(\frac{2}{3}\right)^{n+1} \to 0$, $+1 \to 1$

$\left(\frac{2}{3}\right)^{n} \to 0$, $+1 \to 1$

$3 \cdot \frac{1}{1} = 3$

$$\lim \frac{n^2 - 2n - 1}{n^3 + 1} =$$

$$= \lim \frac{n^2\left(1 - \frac{2}{n} - \frac{1}{n^2}\right)}{n^3\left(1 + \frac{1}{n^3}\right)} =$$

$$= \lim \frac{n^2}{n^3} \cdot \frac{(1 - \text{roba che va a } 0)}{(1 + \text{roba che va a } 0)}$$

$\frac{n^2}{n^3} \to 0$

$$n^2 - n = n^2 \left(1 - \frac{1}{n}\right)$$

$n^2 \to +\infty$, $\left(1 - \frac{1}{n}\right) \to 1$

$$\sqrt{n+1} - \sqrt{n} = \sqrt{n+1}\left(1 - \sqrt{\frac{n}{n+1}}\right)$$

$\sqrt{n+1} \to +\infty$, $\sqrt{\frac{n}{n+1}} \to 1$

$$\frac{(n+1) - n}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{n+1} + \sqrt{n}} \to 0$$

$\sqrt{n+1} + \sqrt{n} \to +\infty$

$$\lim \sqrt[n]{2^n + 3^n} =$$

$1 < H \leq a_n \leq K$

$\sqrt[n]{H} \leq \sqrt[n]{a_n} \leq \sqrt[n]{K}$

$\to 1 \leftarrow$

$\sqrt[n]{p} \to 1 \quad p > 0$

$\sqrt[n]{n} \to 1$

$\sqrt[n]{2^n} \equiv 2$

$$= \sqrt[n]{3^n\left(1+\left(\frac{2}{3}\right)^n\right)} = 3\sqrt[n]{1+\left(\frac{2}{3}\right)^n}$$

$\left(\frac{2}{3}\right)^n \to$ limitata > 1

$\downarrow$

1

$$\lim \sqrt[n]{\frac{n^2+2}{n^2+1}}$$

$$\sqrt[n]{\frac{n^2+2}{n^2+1}} = \sqrt[n]{\frac{n^2}{n^2}\cdot\frac{\left(1+\frac{2}{n^2}\right)}{\left(1+\frac{1}{n^2}\right)}}$$

$n \to \infty$

1

$$\lim \frac{\cos n}{n^2+1}$$

$$0 \le \left|\frac{\cos n}{n^2+1}\right| \le \frac{1}{n^2+1}$$

$\downarrow$ 0 $\qquad$ $\downarrow$ 0

$$\lim n\left(\sqrt{1+\frac{1}{n}}-\sqrt{1-\frac{1}{n}}\right)$$

$$\text{(n)}\ \frac{1+\frac{1}{n}-\left(1-\frac{1}{n}\right)}{\sqrt{1+\frac{1}{n}}+\sqrt{1-\frac{1}{n}}} = n\cdot\frac{\frac{2}{n}}{(\quad\cdot\quad)}$$

$\sqrt{1+\frac{1}{n}} \to 1$, $\sqrt{1-\frac{1}{n}} \to 1$

$(\quad\cdot\quad) \to 2$

$\frac{2}{n} \to 1$

$\frac{1}{2}$

$a_n \to L \Rightarrow \sqrt{a_n} \to \sqrt{L}$

$\forall \varepsilon > 0 \ \exists \nu : \forall n > \nu \quad |\sqrt{a_n} - \sqrt{L}| < \varepsilon$

$$|\sqrt{a_n} - \sqrt{L}| = \frac{|a_n - L|}{\sqrt{a_n} + \sqrt{L}} < |a_n - L| \cdot \frac{1}{\sqrt{L} + \sqrt{L/2}} < \varepsilon$$

$n > \overline{\nu}$; $|a_n - L| < \frac{\varepsilon}{k}$; $\frac{1}{\sqrt{L} + \sqrt{L/2}} < k$

$\overline{\varepsilon} = \frac{L}{2}$

$L - \frac{L}{2} < a_n < L + \frac{L}{2}$

$\frac{L}{2}$

$n > \overline{\nu}$

$\nu_1 : \forall n > \nu_1 \to |a_n - L| < \frac{\varepsilon}{k}$

$\nu = \max\{\overline{\nu}, \nu_1\}$

$n > \overline{\nu}$

$a_n > \frac{L}{2}$

$\Downarrow$

$\sqrt{a_n} > \sqrt{L/2}$

$\lim \sqrt[n]{n!}$

$\lim \frac{k^n}{n!} \to 0 \quad \forall k \in \mathbb{R}^+$

$\left(1+\frac{1}{n}\right)^n \longrightarrow e$

1^∞

$\varepsilon = 1 \quad \left|\frac{k^n}{n!}\right| < \varepsilon \quad \forall n > \bar{\nu} \Rightarrow \frac{k^n}{n!} < \varepsilon = 1$

≥ 0

$\forall k$ a meno di un numero finito

$k^n < n!$

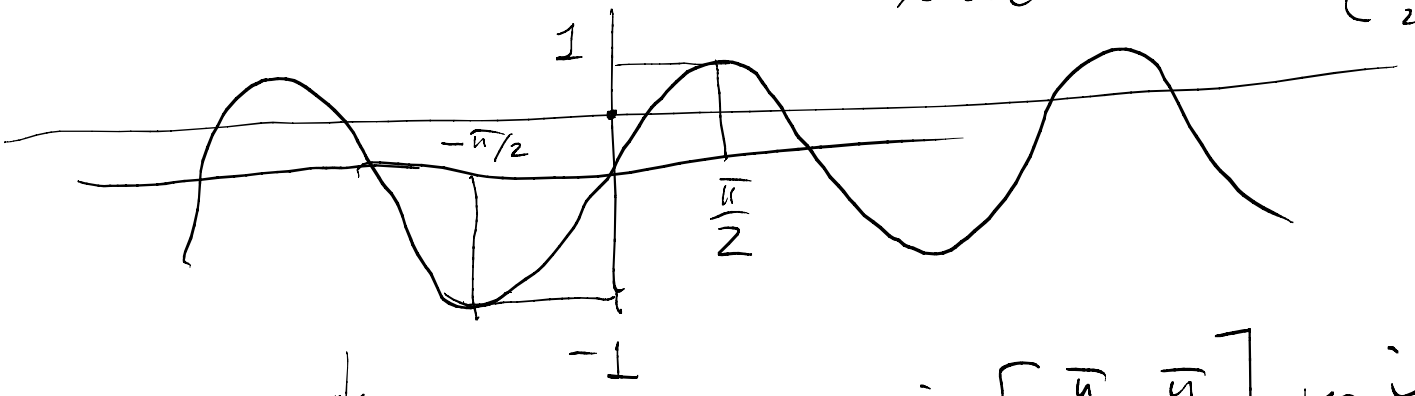
$\forall n > \bar{\nu}$

$\to k < \sqrt[n]{n!}$

$\forall n > \bar{\nu}$

$\frac{d}{dx} \arcsin x$

$\arcsin x$ è l'inversa di $\sin x$ sull'intervallo $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



$\arcsin x$ è l'unico y in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ per il quale $\sin y = x$.

$f(x) = \sin x \qquad f^{-1}(y) = \arcsin y$

$f'(x) = \cos x$

$$\left[f^{-1}(y)\right]' = \frac{1}{f'\left(f^{-1}(y)\right)}$$

$$(\arcsin y)' = \frac{1}{\cos(\arcsin y)}$$

$\sin^2 x + \cos^2 x = 1$

$\cos x = \boxed{?} \sqrt{1 - \sin^2 x}$

$+$

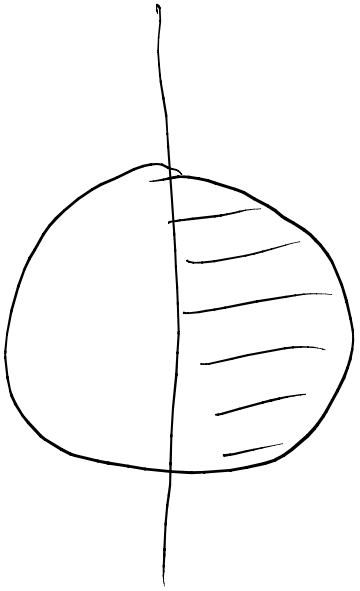
$\cos^2 x = 1 - \sin^2 x$

$|\cos x| = \sqrt{\cos^2 x} = \sqrt{1 - \sin^2 x}$

$\|$

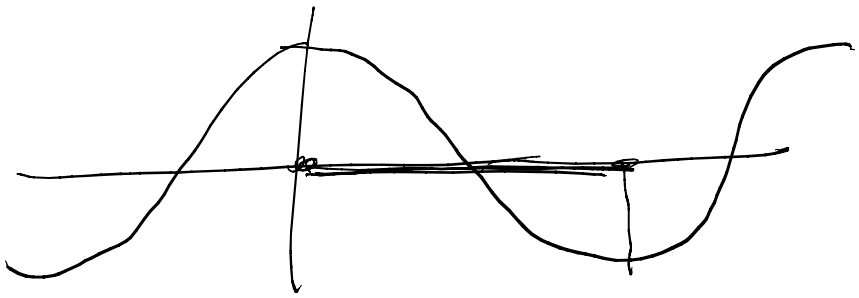
$\cos x$

in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ je $\cos > 0$



$$(\arcsin y)' = \frac{1}{\cos(\arcsin y)} =$$

$$= \frac{1}{\sqrt{1 - \sin^2(\arcsin y)}} = \frac{1}{\sqrt{1 - y^2}}$$



$\arccos y$ è il valore x in $[0,\pi]$ tale che $\cos x = y$.

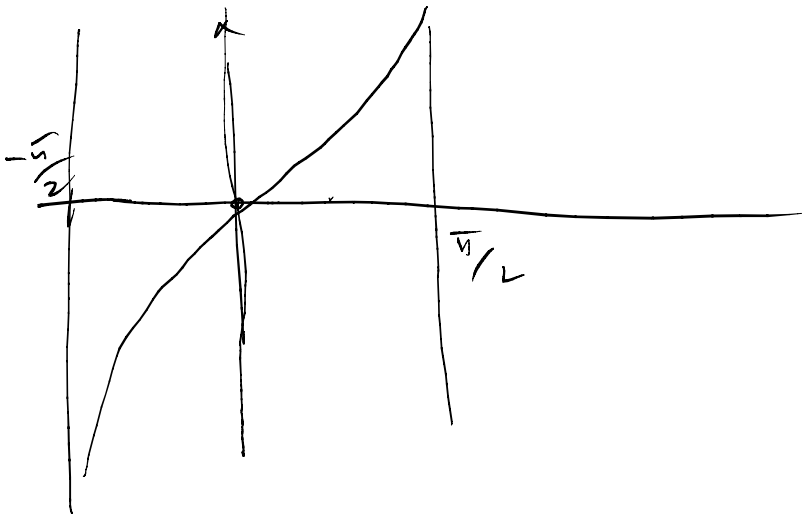
$$\frac{d}{dy}\left[\arccos y\right] = \frac{1}{-\sin(\arccos y)} = \boxed{\frac{1}{-\sqrt{1-y^2}}}$$

$$\sin^2 x = 1 - \cos^2 x$$

$$|\sin x| = \sqrt{1-\cos^2 x}$$

su $[0,\pi]$ $\quad \sin x$ (che è >0)

$$-\frac{1}{\sqrt{1-\cos^2(\arccos y)}}$$



$$\left]-\frac{\pi}{2}, \frac{\pi}{2}\right[$$

$$(\operatorname{tg} x)' = \left(\frac{\sin x}{\cos x}\right)' = \frac{\cos^2 x - \sin x(-\sin x)}{\cos^2 x} =$$

$$= \frac{1}{\cos^2 x} = \frac{\sin^2 x + \cos^2 x}{\cos^2 x} =$$

$$= 1 + \operatorname{tg}^2 x$$

$$(\operatorname{arctg} y)' = \frac{1}{\operatorname{tg}'(\operatorname{arctg} y)} =$$

$$= \cos^2(\operatorname{arctg} y) = \frac{1}{1 + \operatorname{tg}^2(\operatorname{arctg} y)} = \frac{1}{1 + y^2}$$

$$\left[(\sin x)^{\cos x}\right]' = \left[e^{\cos x \lg \sin x}\right]'$$

$>0 \quad \sin x > 0 \quad]0, \pi[\quad + \dots$

$$\left(e^{\cos x \lg \sin x}\right)' =$$

$$= e^{\cos x \lg \sin x} \left(\cos x \lg \sin x\right)' = (\sin x)^{\cos x}$$

la derivata della funzione più esterna e^y con $y = \cos x \lg \sin x$ $(g(x))$

$$(e^x)' = e^x$$

$$(\lg x)' = \frac{1}{x}$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$\left[f(g(x))\right]' = f'(g(x))\, g'(x)$$

ho derivato il prodotto.

$$(\sin x)^{\cos x}\left[-\sin x \lg \sin x + \cos x \,(\lg \sin x)'\right] =$$

ho derivato il lg.

$$= (\sin x)^{\cos x}\left[-\sin x \lg \sin x + \cos x \,\frac{1}{\sin x}\cdot(\sin x)'\right] =$$

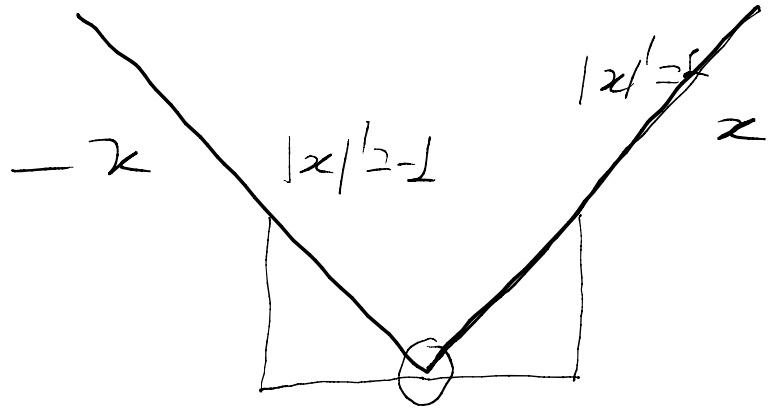
$$\boxed{(x^x)'}$$

$$= (\sin x)^{\cos x}\left[-\sin x \lg \sin x + \cos x \,\frac{\cos x}{\sin x}\right]$$

$\lg|x|$

$$\boxed{\begin{array}{l}(\sqrt{x})' = \dfrac{1}{2\sqrt{x}} \\ (x^2)' = 2x\end{array}}$$

$$|x|' = \left(\sqrt{x^2}\right)' = \frac{1}{2\sqrt{x^2}}\cdot 2x = \frac{x}{|x|}$$



$$|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

$$= \sqrt{x^2} \leftarrow$$

$$\frac{x}{|x|} = \begin{cases} 1 & x > 0 \\ -1 & x < 0 \end{cases}$$

NON E' DET. per $x = 0$.

$$x = |x| \frac{x}{|x|}$$

$$\operatorname{sgn} x = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

$$\left(\ln|x|\right)' = \frac{1}{|x|}\,\frac{x}{|x|} = \qquad \frac{x}{|x|} = \begin{cases} 1 & x>0 \\ -1 & x<0 \end{cases}$$

$$= \frac{1}{|x|}\,\frac{|x|}{x} \underset{x\neq 0}{=\!=} \frac{1}{x} \qquad \frac{|x|}{x} =$$

$$\left(\ln|x|\right)' = \frac{1}{x} \qquad f \to f'$$

f $\exists F$: $F' \equiv f$

$(\cos t)'$ PRIMITIVA $(F+\cos t)' = F' + (\cos t)' = F' = f$