Note Title 10/13/2021

il limite della somma è la somma dei limiti

di differenza
rapporto
prodotto
reciproco

differenza
rapporto
prodotto
reciproco

se il denominatore ha limite non nullo

$$\frac{(-1)^n \frac{1}{n}}{\frac{1}{n}} = (-1)^n \quad \text{oscilla}$$

$$\frac{(-1)^n \frac{1}{n}}{\frac{1}{n^2}} \equiv (-1)^n n$$

$\frac{a_n}{b_n}$ $\quad a_n \to L \quad L \neq 0$

$b_n \to 0 \quad b_n > 0$

$$\lim \frac{a_n}{b_n} = \begin{cases} +\infty & \text{se } L > 0 \\ -\infty & \text{se } L < 0 \end{cases}$$

$a_n \cdot \frac{1}{b_n}$

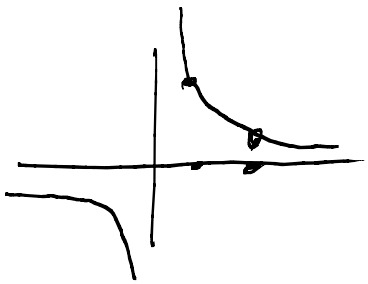
$a_n \to L$

$\lim \frac{1}{b_n}$

$b_n \to 0 \qquad \frac{1}{|b_n|} \to +\infty$

$L - \varepsilon < \quad < L + \varepsilon$

$n > \nu$



$\frac{1}{|b_n|} > \varepsilon \iff |b_n| < \frac{1}{\varepsilon}$

vera $\forall n > \nu$ perché $b_n \to 0$

$\varepsilon = \frac{L}{2}$ $\quad a_n > L - \frac{L}{2} \quad \forall n > \bar{\nu}$ ←

$= L/2$

$a_n \cdot \frac{1}{b_n} > \frac{L}{2} \cdot \frac{1}{b_n} > \varepsilon$ ↓ vale $\forall n > \nu_2$

$\frac{1}{b_n} > \frac{2\varepsilon}{L} \qquad 0 < b_n < \frac{L}{2\varepsilon}$

$\nu = \max\{\bar{\nu}, \nu_2\}$

$\frac{1}{(-1)^n \frac{1}{n}} = (-1)^n n$

$-1, 2, -3, 4, \dots, (-1)^n n$

ε

$\boxed{a_n \to +\infty}$ $\boxed{b_n \geq H \ \forall n \in \mathbb{N}}$

$a_n + b_n \geq a_n + H > \varepsilon$

$a_n + H > \varepsilon$

$\boxed{a_n > \varepsilon - H}$ è verificata $\forall n > \nu$

è l'ε da ~~usare~~ nell'ipotesi di divergenza di a_n

$\boxed{\varepsilon}$ della tesi

$\lim a_n + b_n = +\infty$

$b_n \to 0 \quad b_n < 0 \qquad \frac{1}{b_n} = -\frac{1}{|b_n|} \to -\infty$

$\frac{1}{|b_n|} \to +\infty$

$a_n \to L > 0$
$b_n \to +\infty$
$\Rightarrow a_n b_n \to +\infty$

$\varepsilon = L/2$

$L - \varepsilon$

$a_n > L - \frac{L}{2} = \frac{L}{2} > 0$ $\quad \forall n > \nu_1$

$a_n b_n > \frac{L}{2} b_n > \varepsilon$ della tesi

$n > \nu_2$

$b_n > \frac{2\varepsilon}{L}$

$\forall \sigma > 0 \ \exists \nu_2 : \forall n > \nu_2 \ b_n > \sigma$

$\sigma = \frac{2\varepsilon}{L}$ → della tesi

$\forall n > \nu$ fornito dalla disuguaglianza di b_n

$\nu = \max\{\nu_1, \nu_2\}$

$a_n b_n$

$a_n \to 0$

$b_n \to +\infty$

$\begin{cases} a_n = \frac{1}{n} \\ b_n = n^2 \end{cases} \Rightarrow a_n b_n = n \to +\infty$

$\begin{cases} a_n = (-1)^n \frac{1}{n} \\ b_n = n^2 \end{cases} \Rightarrow a_n b_n = (-1)^n n$ oscill.

$\begin{cases} a_n = \frac{1}{n} \\ b_n = n \end{cases} \Rightarrow a_n b_n = 1 \to 1$

$\begin{cases} a_n = \frac{1}{n^2} \\ b_n = n \end{cases} \Rightarrow a_n b_n = \frac{1}{n} \to 0$

$a_n \to +\infty$

$b_n \to +\infty$

$\frac{a_n}{b_n} \to ?$

$\begin{cases} a_n = n^2 \\ b_n = n \end{cases} \quad \frac{a_n}{b_n} = n \to +\infty$

$\begin{cases} a_n = 2n \\ b_n = n \end{cases} \quad \frac{a_n}{b_n} = 2 \to 2$

$\begin{cases} a_n = n \\ b_n = n^2 \end{cases} \quad \frac{a_n}{b_n} = \frac{1}{n} \to 0$

$a_n \to +\infty$

$b_n \to +\infty$

$a_n - b_n$

$a_n = n$

$b_n = n+1$

$a_n - b_n \equiv -1 \to -1$

$a_n = n^2$

$b_n = n$

$a_n - b_n$?

$$a_n - b_n = n^2 - n = n^2\left(1 - \frac{n}{n^2}\right) \longrightarrow +\infty$$

$\frac{n}{n^2} \to 1$

$$\sqrt{n+1} - \sqrt{n} =$$

$$= \frac{(\sqrt{n+1} - \sqrt{n})(\sqrt{n+1} + \sqrt{n})}{\sqrt{n+1} + \sqrt{n}} =$$

$$= \frac{(n+1) - n}{\sqrt{n+1} + \sqrt{n}} = \frac{1}{\sqrt{n+1} + \sqrt{n}}$$

$\sqrt{n+1} + \sqrt{n} \to +\infty$, $\frac{1}{\to +\infty} \to 0$

$\sqrt{n} \to +\infty$

$\sqrt{n} > \varepsilon \iff n > \varepsilon^2$

$\sqrt{n+1} \to +\infty$

$\sqrt{n+1} > \varepsilon \quad n+1 > \varepsilon^2 \Leftarrow n > \underline{\varepsilon^2 + 1}$

Forme indeterminate

$\frac{0}{0}$, $\frac{\infty}{\infty}$, $0 \cdot \infty$, $\infty - \infty$

$a_n^{b_n} = e^{b_n \lg a_n}$

$a^b = e^{b \lg a}$

$b_n \to 0$

$\lg a_n \to +\infty$ oppure $-\infty$

$a_n \to +\infty$

$a_n \to 0$

$b_n \to +\infty$ $-\infty$

$\lg a_n \to 0$

$e_n \to 1$

$0^0, \infty^0, 1^\infty$

$\frac{f(x) - f(x_0)}{x - x_0}$