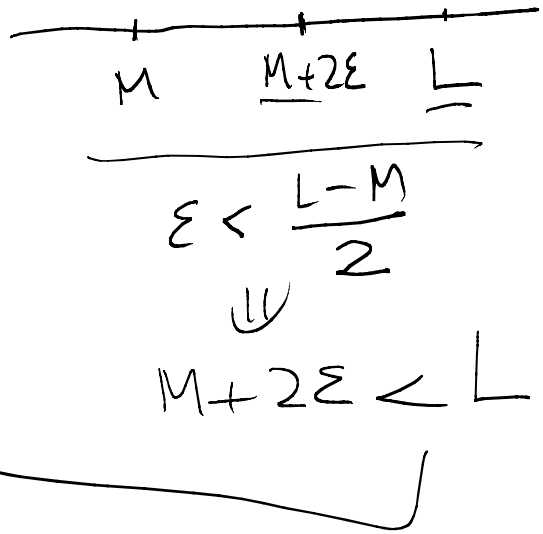
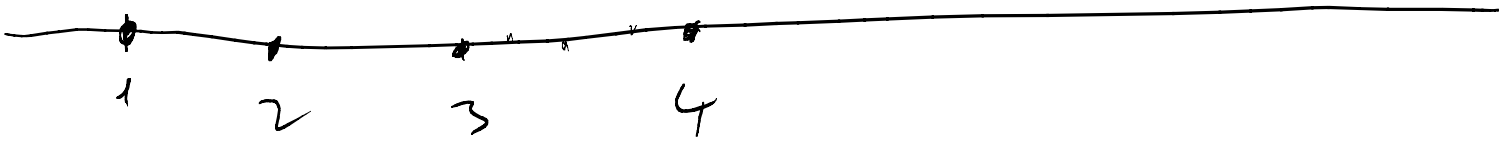
Note Title 10/12/2021

$\rightarrow$ $L < M + 2\varepsilon \quad \forall \varepsilon > 0$ $\Rightarrow$ $L \leq M$

Per assurdo, se fosse falso, sarebbe vero $M < L$

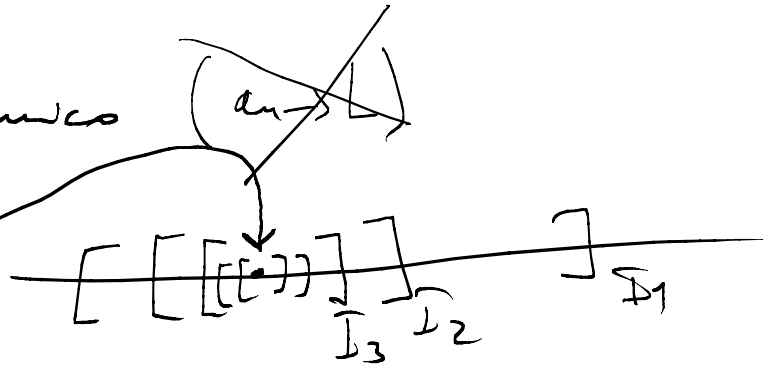




$I_n = [a_n, b_n]$ $\quad |I_n| = (b_n - a_n) < 10^{-n}$ $\quad |I_n| \to 0$

$I_n \supseteq I_{n+1}$

$\exists\, L \in I_n \ \forall n \in \mathbb{N}$, único $\quad (a_n \to L)$



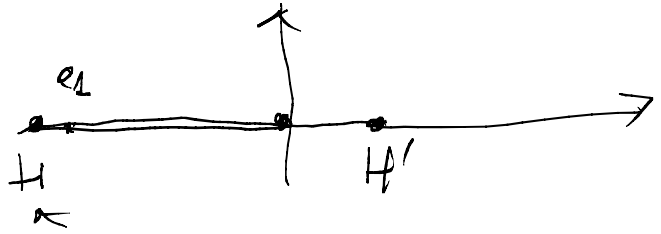
$a_n \to L \Rightarrow \exists k : |a_n| \le k \ \forall n$ $\quad -k \le a_n \le k$

$\varepsilon = 1$ $\quad \exists \nu : \forall n > \nu$ $\quad L - 1 < a_n < L + 1$

$$\min\{a_1, a_2 \dots a_\nu, L-1\} \le a_n \le \max\{a_1, a_2, \dots a_\nu, L+1\}$$

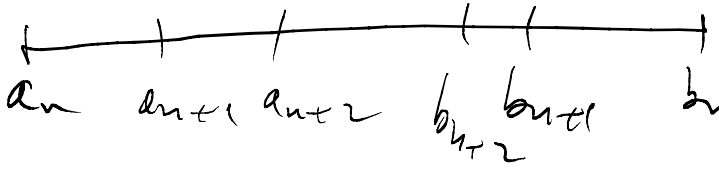
H $\qquad$ H'

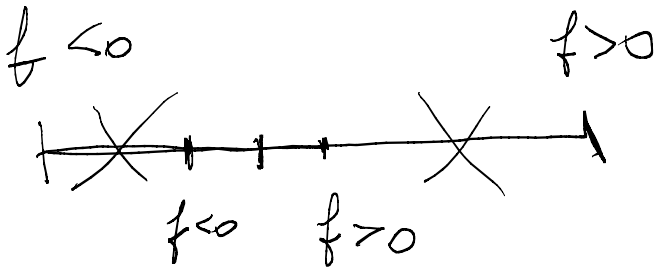
$$a_n \in [H, H']$$



k

$$k = \max\{|H|, |H'|\}$$





$x^* = \sup a_n$ ↑ $\quad y^* = \inf b_n$ ↓

$$\boxed{x^* = y^* = \lim a_n = \lim b_n}$$

$$\boxed{a_n \leq x^* = y^* \leq b_n} \quad \forall n$$

$$\boxed{|b_n - a_n| = 10^{-n}}$$

7.5 7.55
7.6

$L \le M$

$L < M + 2\varepsilon \quad \forall \varepsilon > 0$

$M < L \qquad 2\varepsilon < L - M$

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a_n limitata $\Leftrightarrow \exists k : |a_n| \le k \ \forall n \in \mathbb{N}$

b_n infinitesima $\Leftrightarrow \forall \varepsilon > 0 \ \exists \nu : \forall n > \nu \ |b_n| < \varepsilon$

$\exists$ $\lim_{n \to L} a_n b_n = 0$ cioè $\forall \varepsilon > 0 \ \exists \nu : \forall n > \nu \ |a_n b_n - 0| < \varepsilon$

$|a_n b_n| = |a_n||b_n| \le k|b_n| < \varepsilon$ vera se $|b_n| < \frac{\varepsilon}{k}$

($k|b_n| < \varepsilon \Rightarrow |b_n| < \frac{\varepsilon}{k}$)

TS $\lim a_n b_n = 0$

$\forall \varepsilon > 0 \ \exists \nu : \forall n > \nu \ |a_n b_n| < \varepsilon$ da fare

Io decido a te l'ε che vuoi.

Prendo la seconda ipotesi e la divido per k. Con $\frac{\varepsilon}{k}$ vado alla seconda ipotesi con questo numero $\exists \nu : \forall n > \nu$

$|b_n| < \frac{\varepsilon}{k}$ $\quad |a_n||b_n| < k \frac{\varepsilon}{k} = \varepsilon$

$|a+b| \le |a|+|b|$

$a = a-b+b$

$|a| \le |a-b|+|b| \Rightarrow \boxed{|a|-|b| \le |a-b|}$

$b = b-a+a \Rightarrow |b| \le |b-a|+|a| \Rightarrow \boxed{|b|-|a| \le |b-a|}$

sono uguali

$\boxed{\big||a|-|b|\big| \le |a-b|}$

$a_n \to L$

$\big||a_n|-|L|\big| \le |a_n - L|$

$|a_n| \to |L|$ $\quad |a_n - L| \to 0$ $\quad a_n \to L$

$$\left|\frac{1}{a_n} - \frac{1}{L}\right| = \frac{|L - a_n|}{|a_n||L|} \leq$$

$|a_n| \to |L| \neq 0$

$\varepsilon = \frac{|L|}{2}$

$$\leq \frac{|L - a_n|}{\frac{|L|}{2}|L|} = \frac{2}{|L|^2}\underbrace{|L - a_n|}_{< \varepsilon} \qquad \frac{|L|}{2} = |L| - \frac{|L|}{2} < |a_n| <$$

$\varepsilon \frac{|L|^2}{2}$

Perché Eudosso-Archimede non vale in cui accade assumere che $a = 2.412\overline{9} = 2.413\overline{0} = b$

Se fosse $a \neq b$ allora $b - a > 0$

$\forall a, b > 0 \quad \exists n : na > b$

$b - a < 10^{-n} \quad \forall n$

$10^n (b - a) < 1 \quad \forall n$