Note Title 10/5/2021

Prova per induzione

$2^n \geq n+1$ $P(n)$

$P(1) \equiv "2^1 \geq 1+1"$

1) $P(1)$ è vera $\quad 2^1 = 1+1$ $\quad n=1$

2) $P(n) \Rightarrow P(n+1)$ $\forall n \in \mathbb{N}$

$P(n)$: si suppone vera

$P(n+1)$: si dimostra a partire da

$P(n)$

$2^n \geq n+1$

$P(n+1)$

$2^{n+1} \geq (n+1)+1$

$P(n)$

$\geq$

$2^{n+1} = 2 \cdot 2^n \geq 2(n+1) = 2n+2 > n+2$

$P(n+1) \Leftarrow 2^{n+1} \geq n+2$

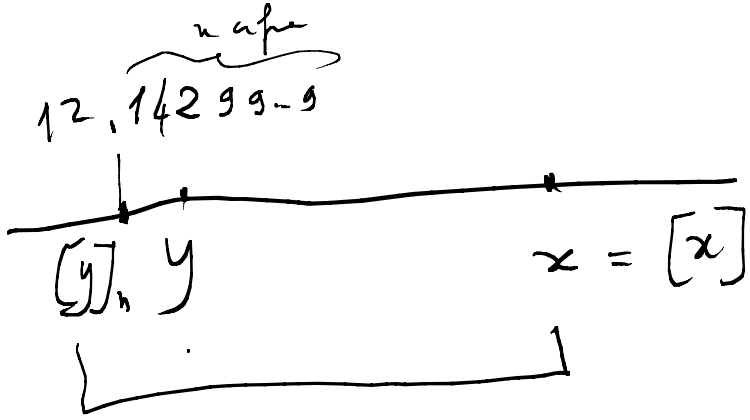
decimale finito $x = \boxed{12.143}\,\overline{0}$

$y = 12.142\overline{9}$

si devono considerare identici

$12.\overbrace{14299\ldots9}^{n \text{ cifre}}$

$[y]_n \quad y \qquad x = [x]_n$ non appena $n \geq 3$



$$[y]_n < y \leq x = 12.143$$

$$\begin{array}{r} 12.\overbrace{1430\ldots0}^{n \text{ cifre}} \\ 12.14299\ldots9 \\ \hline 000000\ldots01 \end{array} = 10^{-n}$$

se $x \neq y$ $\quad 0 < \boxed{x-y} < x - [y]_n = 10^{-n}$

$a < b \equiv$ la stessa cosa $\boxed{a \neq b}$ $\boxed{\underline{e}}$ $a \leq b$

$$\begin{cases} a \neq b \\ a \leq b \end{cases} \Longrightarrow \begin{array}{l} a \neq b \\ [a]_n \leq [b]_n \ \forall n \end{array}$$

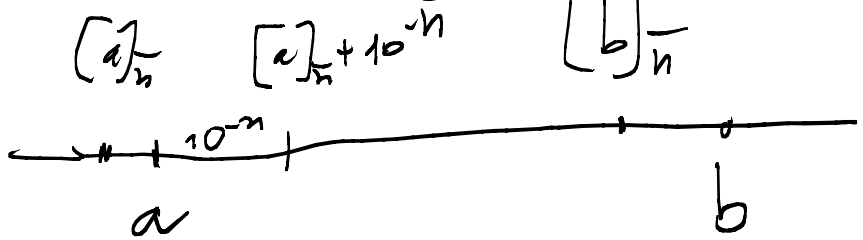
$\forall\, \underline{a < b}$, $a, b \in \mathbb{R}$ $\exists\, q$ decimale finito

$\Downarrow$

$a \neq b$

$a \leq q \leq b$

$\exists\, \bar{n}$: $[b]_{\bar{n}} - [a]_{\bar{n}} \geq \underline{2 \cdot 10^{-\bar{n}}}$



Sia $[a]_{\bar{n}} + 10^{-n}$

Sia $[b]_{\bar{n}}$ separano a e b

$f: X \to Y$ si dice INIETTIVA se $f(x) = f(x') \Rightarrow x = x'$ $\forall x, x' \in X$

2) agli elementi distinti del dominio corrispondono valori distinti delle immagini

3) $x \neq x' \Rightarrow f(x) \neq f(x')$ $\forall x, x' \in X$

Meglio:
Per ogni coppia di elementi distinti del dominio le loro immagini sono distinte

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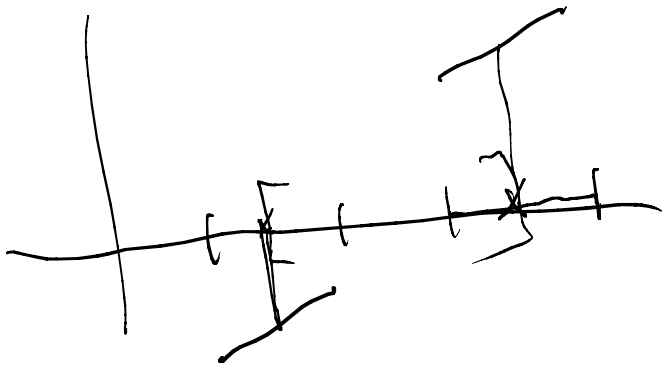
$f: X \to Y$ si dice SURIETTIVA se $\forall y \in Y \; \exists x \in X : y = f(x)$

cioè $\text{cod} f = \text{Im} f$



$a \neq b$ è l'opposto (il NOT, la negazione) di $a = b$

$a = b \equiv$ $\boxed{\forall n \in \mathbb{N} \quad |[a]_n - [b]_n| \leq 10^{-n}}$

è' FALSA SE

$$\exists \bar{n} \in \mathbb{N} \quad |[a]_{\bar{n}} - [b]_{\bar{n}}| > 10^{-\bar{n}}$$

oppure $\geq 2 \cdot 10^{-\bar{n}}$

$10^{-\bar{n}} = 0.000\ldots01$

$2 \cdot 10^{-\bar{n}} = 0.000\ldots02$ il primo disp. $> 10^{-\bar{n}}$

$1.\overline{0} \equiv \boxed{0.\overline{9}}$

$1.\overline{0} = 1.0\ldots0$

$0.\overline{9} > 0.9\ldots9$ (n cifre)

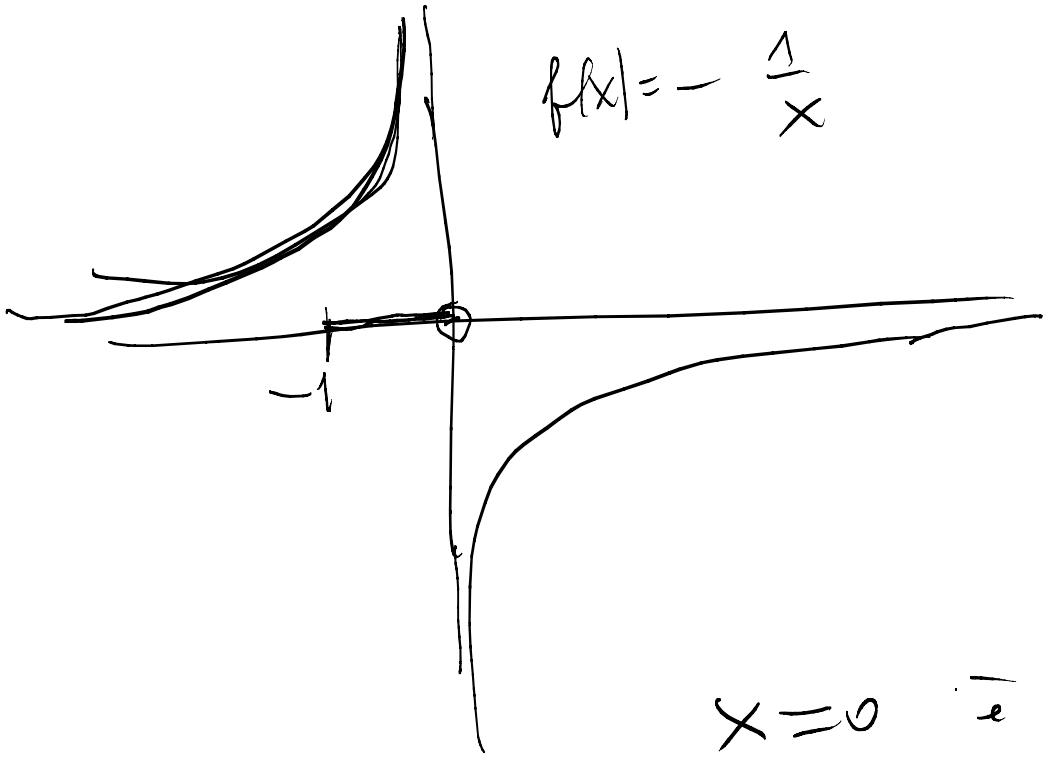
$1.000\ldots0\ldots$

$N = 1 \quad a_i = 0 \ \forall i \in \mathbb{N}$

$f : X \to Y$

$\mathrm{Im} f = f(X) = \{y \in Y : \exists x \in X \ \ y = f(x)\}$

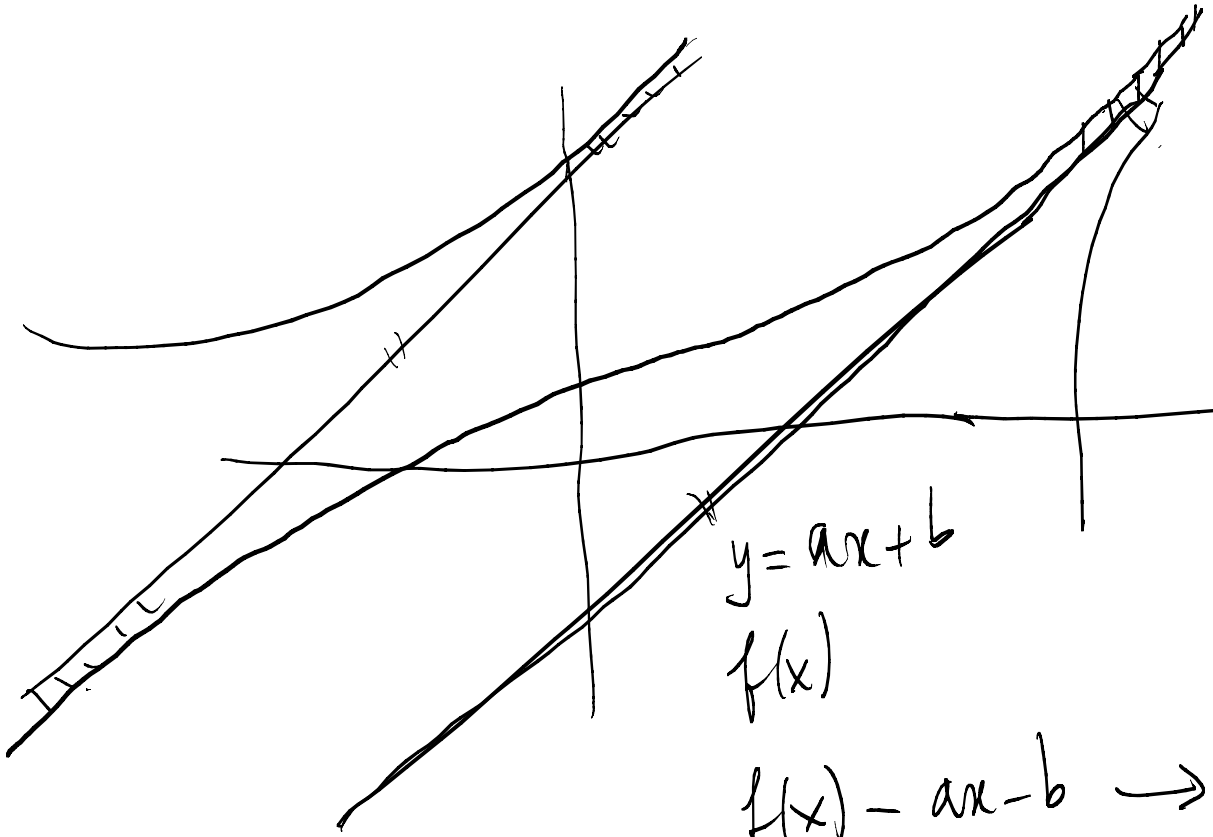
(sinonimi)



$y = 0$ è asintoto orizzontale tanto a $+\infty$ quanto a $-\infty$.

$x = 0$ è un asintoto verticale in 0 tanto in $]-1, 0[$ quanto in $]0, 1[$

$$\lim_{x \to 0^-} f(x) = +\infty \qquad \lim_{x \to 0^+} f(x) = -\infty$$



$y = ax + b$

$f(x)$

$f(x) - ax - b \to 0$

4 comportamenti asintotici del grafico rispetto a 2 rette oblique

$N + \sum a_i 10^{-i}$.

$\frac{3}{10} + \frac{7}{100} + \frac{5}{1000} \ldots$

$0.375\ldots$

$\frac{a_1}{10} + \frac{a_2}{100} + \frac{a_3}{1000} + \ldots + \frac{a_n}{10^n} + \ldots$ dopo la virgola

$N + \sum_{1}^{\infty} \frac{a_k}{10^k}$ → cifre

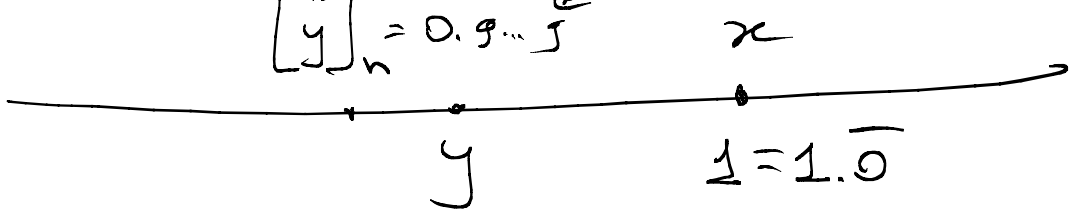
serie decimale

parte intera

$x = 1.00\ldots0\ldots = 1.\overline{0}$

$y = 0.99\ldots9\ldots = 0.\overline{9}$

$$|1 - 0.\underbrace{99\ldots9}_{n}| = 10^{-n}$$



$x \neq y \Leftrightarrow \boxed{x - y \neq 0}$ $\quad x > y \Rightarrow \boxed{0 < x - y}$

$0 < x - y$ $\quad \boxed{x - y} \leq 10^{-n}\ \forall n$ $\quad 10^{n}(x - y) \leq 1\ \forall n$

$\boxed{x-y}$: >0

10^{n}: $=$

$(x-y)$: $\uparrow$