Note Title 10/5/2021

SERIE DECIMALI

$10 \,|\, 3$
$1 \quad 0{,}3\ldots$

$0.33\ldots3\ldots$

$$3\cdot10^{-1} + 3\cdot10^{-2} + \ldots + 3\cdot10^{-n} + \ldots$$

$$= \boxed{\sum_{n=1}^{\infty} 3\cdot10^{-n}} \equiv \boxed{\lim} \sum_{k=1}^{n} 3\cdot10^{-k}$$

$N \in \mathbb{Z}$

$$x = N.\,a_1 a_2 \ldots a_n \ldots$$

$a_i \in \{0,1,2,\ldots,9\}$

$[x]_n = N.\,a_1 a_2 \ldots a_n$ solo le prime n cifre decimali

troncato (per difetto) di x a n cifre.

$$[x]^n = [x]_n + 10^{-n}$$

$$[x]_n \leq x \leq \boxed{[x]_n + 10^{-n}}$$

$$[x]^n$$

$$\pi = 3.1415\ldots$$

$$[\pi]_2 = 3.14$$

$$[\pi]_2 = 3.15$$

$$x \longrightarrow [x]_0 \equiv [x]$$

$N.a_1 \dots a_n 000000\dots0\dots \equiv N.a_1 \dots a_n \overline{0}$

decimali finiti

$\frac{m}{n}$

$m = nq + r \qquad r < n$

$N.a_1 a_2 \dots a_n \overline{a_{n+1} \dots a_{n+p}}$

$a_{n+1} \dots a_{n+p}$ si ripetono all'infinito ed è detto PERIODO

$x = 0.\overline{a_1 \dots a_n} = 0.a_1 a_2 \dots a_n \overline{a_1 \dots a_n}$

$10^n x = a_1 a_2 \dots a_n . \overline{a_1 a_2 \dots a_n}$

$(10^n - 1)x = a_1 \dots a_n$

$$x = \frac{a_1 \dots a_n}{\underbrace{99\dots9}_{n \text{ cifre}}}$$

$0.1\,01\,001\,000\,1\,0000$

$31.12\overline{347} = 31.12 + 0.00\overline{347}$

$(\mathbb{Q}, 9)$ $\frac{p}{q}$. $\frac{3\,12}{100} + \frac{347}{999 \cdot 100}$

$0.\overline{9} = 0.999\ldots9\ldots$

$\frac{9}{9} = 1$

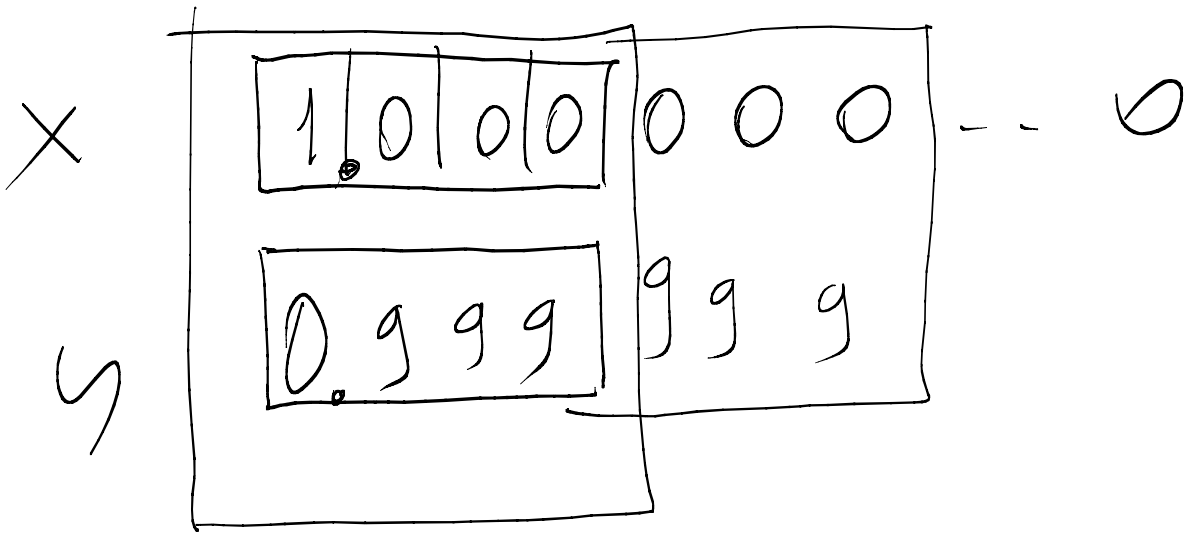
$0.\overline{3} + 0.\overline{6} = 0.\overline{9}$

$\frac{3}{9} + \frac{6}{9} = 1$

$0.33\ldots3\ldots +$

$0.66\ldots6\ldots$

$0.\overline{9} = 1.\overline{0}$?

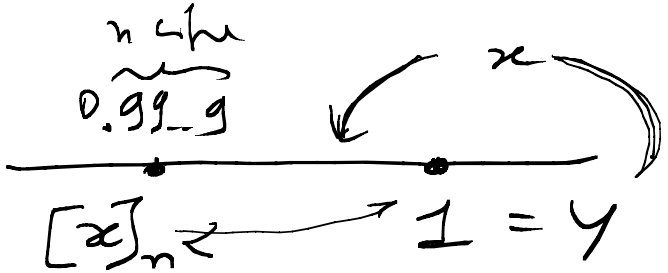


$$|x-y| \leq 10^{-3}$$

$x \quad 0.\overline{9}$

$y \quad 1.\overline{0}$

$$[y]_n - [x]_n = 10^{-n}$$



$\mathbb{R}$ è $\forall x, y > 0 \quad \exists n : nx > y$

EUDOSSO – ARCHIMEDE

$$n \frac{p}{q} > \frac{p'}{q'} \iff n > \frac{p'q}{pq'}$$

$p > 0$ $\quad$ $p' > 0$

$$x \leq 10^{-n} \quad \forall n$$

se $x \geq 0$ allora $x = 0$

il problema

ANALISI NON STANDARD (EUDOSSO FACILE)

Un numero reale è una serie decimale, ove vengono identificate le serie decimali finite con quelle di periodo 9 corrispondenti

$$N, a_1 \dots a_n \overline{0} \equiv N, a_1 \dots (a_n - 1)\overline{9}$$

lo stesso reale

$$3,14100\dots0\dots \equiv 3,14099\dots9\dots$$

$1,418000\dots0\dots$ identico a $1,41799\dots9\dots$

$a_3 = 8$ $\qquad$ $a_3 - 1$

$$a = b \iff |[a]_n - [b]_n| \leq 10^{-n} \quad \forall n \in \mathbb{N}$$

$$1.0000 - 0.9999 = 10^{-4}$$

$$a \neq b \iff \exists \bar{n} : |[a]_{\bar{n}} - [b]_{\bar{n}}| > 10^{-n}$$

$$\Downarrow$$

ovvero

$$\exists \bar{n} : |[a]_{\bar{n}} - [b]_{\bar{n}}| \geq 2 \cdot 10^{-n}$$

$a \leq b$ o $a = b$ ovvero $[a]_n \leq [b]_n \quad \forall n \in \mathbb{N}$

$a = 1.\bar{0} \qquad b = 0.\bar{9}$

$a = b$ ma $[a]_n > [b]_n$

criterio $\Rightarrow$ Se $\forall n \in \mathbb{N} \quad [a]_n \leq [b]_n$

$\Downarrow$

allora $a \leq b$

$a < b$ $\quad\Rightarrow\quad$ $a \neq b$ e $a \leq b$

$[x]_2 = [y]_2$

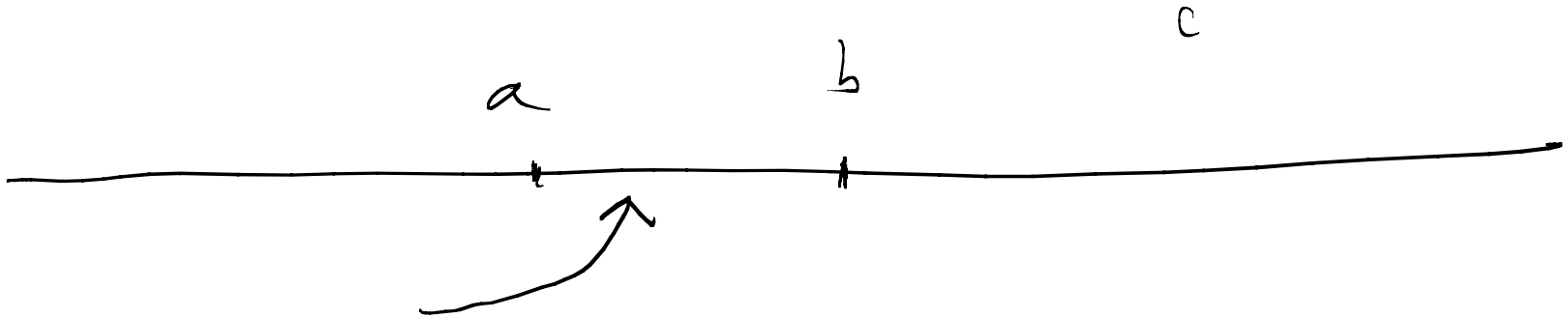
$[x]_3 < [y]_3$

$x = 1.213\overline{40}$ $\qquad$ $y = 1.21\overline{412}7$

$x \neq y$ perché $\left| [x]_4 - [y]_4 \right| = 7 \cdot 10^{-4} > 2 \cdot 10^{-4}$

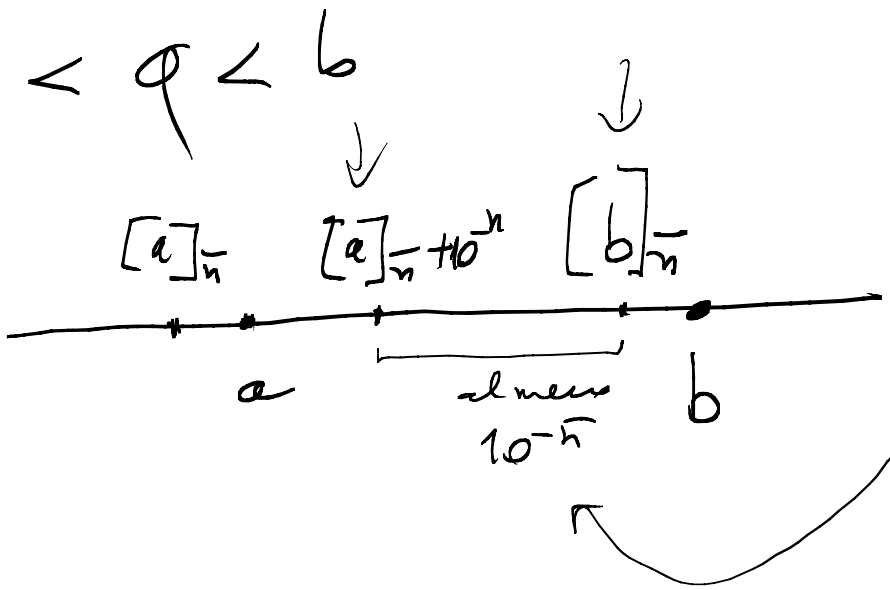
mentre $[x]_n \leq [y]_n$ $\forall n$

Densità dei decimali finiti



Se $a < b$ $\exists$ q decimale finito tale che

$$a < q < b$$

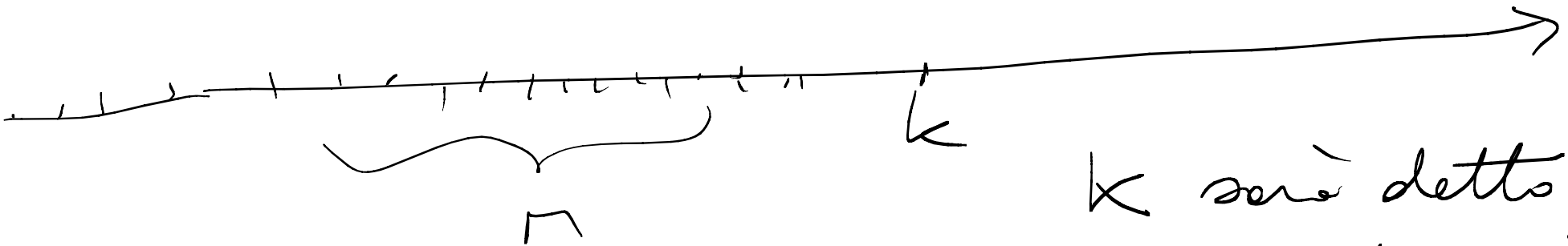


$$[b]_{\frac{1}{n}} - [a]_{\frac{1}{n}} \geq 2 \cdot 10^{-n}$$

$$q = [a]_{\frac{1}{n}} + 10^{-n}$$

$M \subseteq \mathbb{R}$ si dice limitato superiormente se

$$\exists k \in \mathbb{R} : \quad x \leq k \quad \forall x \in M$$

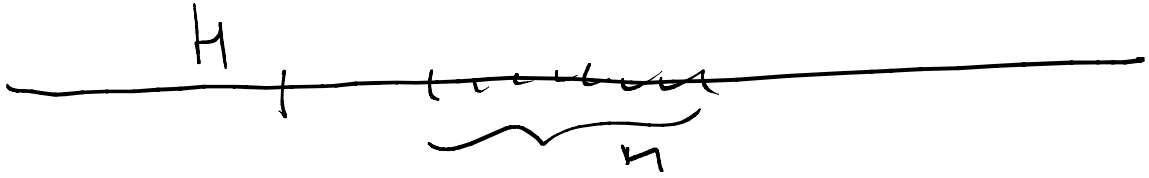


k sarà detto Maggiorante per M

$M \subseteq \mathbb{R}$

M si dice limitato inferiormente se $\exists H \in \mathbb{R}$:

$$x \geq H \quad \forall x \in M$$



H sarà detto minorante.

$M = [-1, 1[$

$k = 2$ è un magg.

$k = 1$ è un maggiorante

$H = -1$ è un minorante

$H = -18$ è un minorante

M è detto <u>limitato</u> se è limitato inferiormente e superiormente $\exists H, K \in \mathbb{R} : H \leq x \leq K \quad \forall x \in M$.

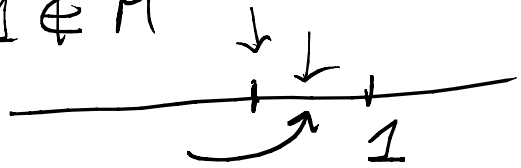
$x_0 \in \mathbb{R}$ è detto <u>massimo</u> di M se verifica

1) $x_0 \in M$

2) $x_0 \geq x \quad \forall x \in M$

$M =]-1, 1[$ NON HA MASSIMO

$1 \notin M$



x_0 è detto <u>minimo</u> di M se

1) $x_0 \in M$

2) $x_0 \leq x \quad \forall x \in M$

$]-1, 1[$ $x_0 = -1$ è minimo