Note Title 10/1/2021

$f(x) = x^2 \qquad f: \mathbb{R}_0^+ \longrightarrow \mathbb{R}_0^+$

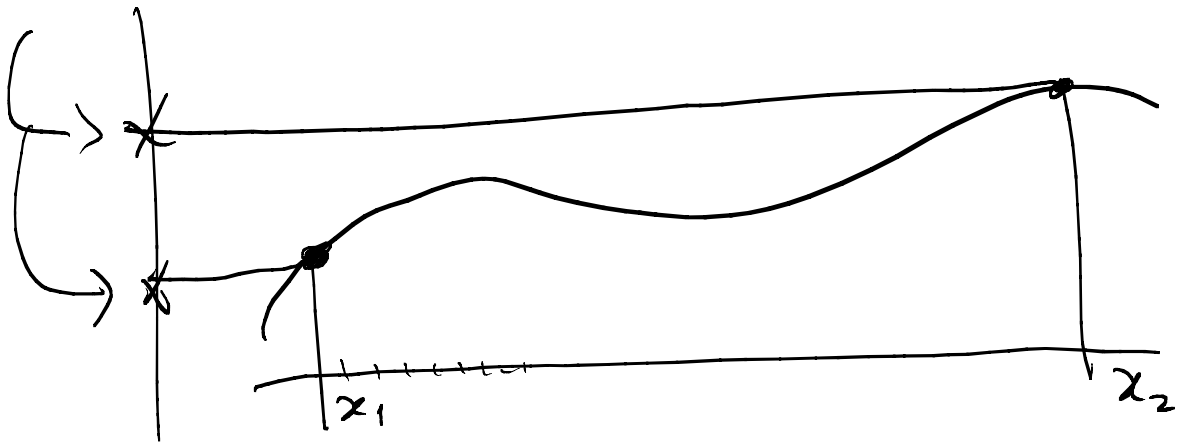
Th. dei valori intermedi (Bernard Bolzano) Weierstrass

$f: A \to \mathbb{R}$, $A \subseteq \mathbb{R}$

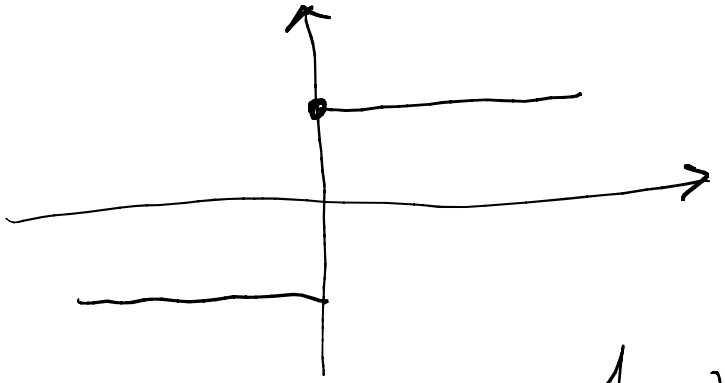
1) A sia un intervallo

→ 2) f sia continua

Ts. f assume tutti i valori compresi fra quelli assunti in due punti arbitrari del dominio



se $f: [x_1, x_2] \to \mathbb{R}$
ed è continua
allora assume
tutti i valori compresi
tra $f(x_1)$ e $f(x_2)$



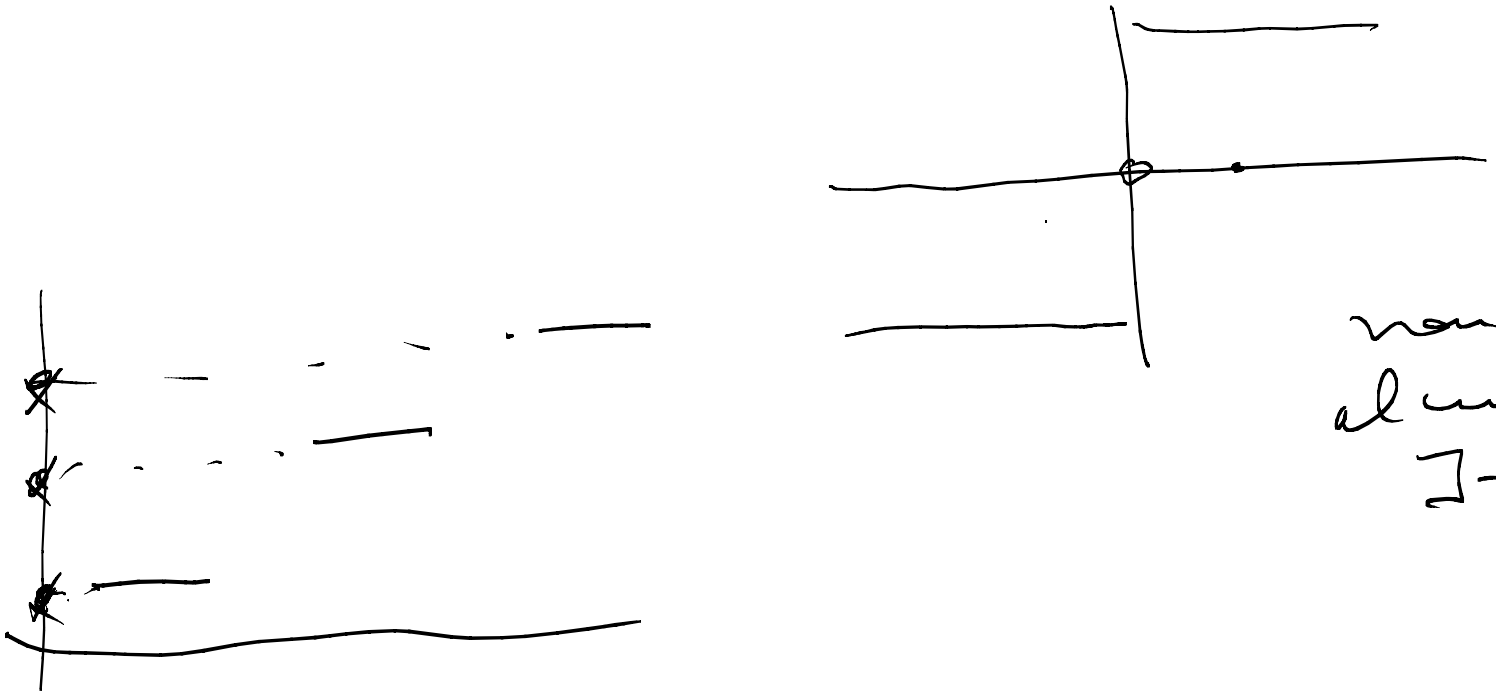
$$f(x) = \begin{cases} -1 & x < 0 \\ 1 & x \geq 0 \end{cases}$$

f non assume alcun valore
dell'intervallo $]-1, 1[$
(estremi esclusi)

$$f(x) = \begin{cases} -1 & x < 0 \\ 1 & x > 0 \end{cases}$$

$\frac{x}{|x|}$

dove $\{x \neq 0\}$



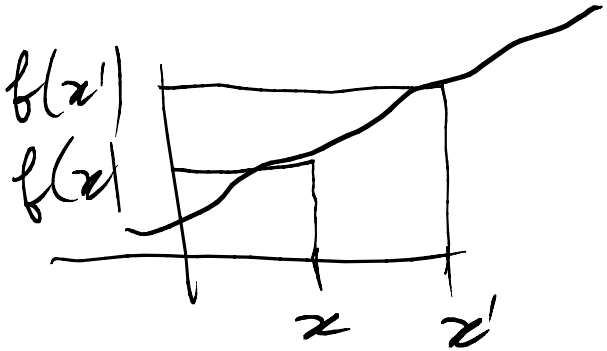
non assume alcun valore in $]-1, 1[$

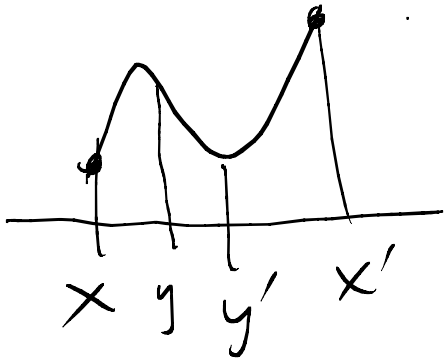
$$\left|(x_0+h)^2 - x_0^2\right| = \left|2x_0h + h^2\right| = |h|\,\underbrace{\left(|2x_0+h|\right)}_{\simeq\, 2x_0} \le$$

$$\le \left[\,|2x_0|+1\,\right]|h|$$

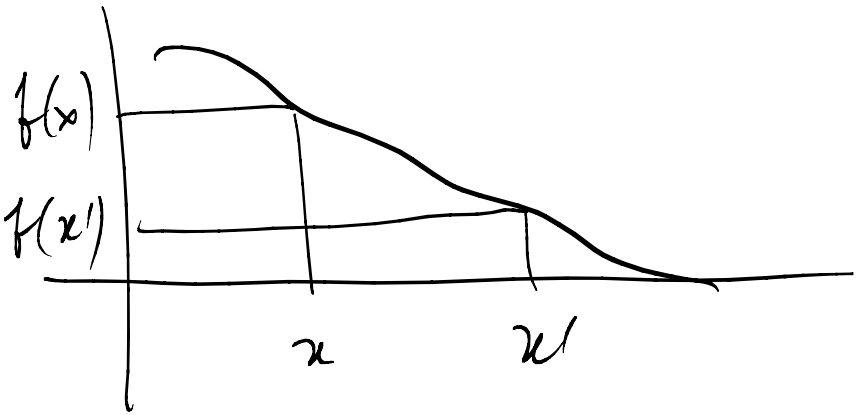
$f : X \to Y \qquad X, Y \subseteq \mathbb{R}$

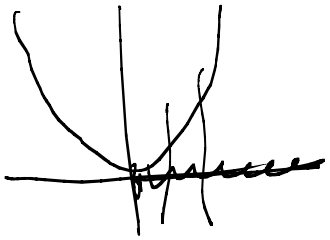
f crescente in X se e solo se $\boxed{\forall x, x' \in X \quad x \le x' \Longrightarrow f(x) \le f(x')}$





f è decrescente se $\forall x, x' \in \operatorname{dom} f \quad x \le x' \Longrightarrow f(x) \ge f(x')$

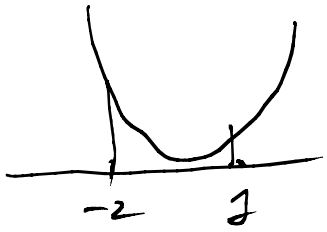




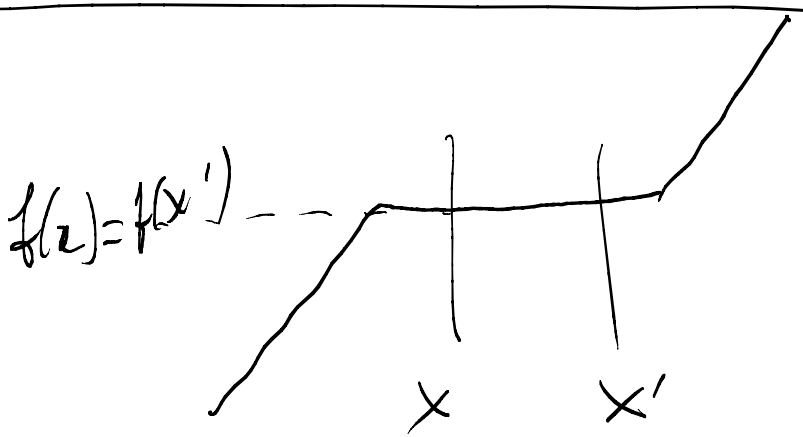
$$\underbrace{\sqrt{x}}_{\geq 0} \leq x \qquad x \geq 0$$
$$\geq 0$$

$$x \leq x^2$$

$$(-2)^2 \qquad 1^2$$



$$? \uparrow \quad a \leq b \quad \downarrow ?$$
$$a^2 \leq b^2$$



$$x < x' \qquad f(x) = f(x')$$

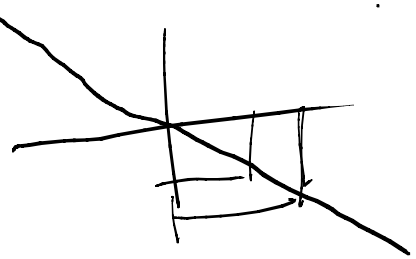
f è detta STRETTAMENTE CRESCENTE

$\forall x, x' \in \text{dom } f \quad x < x' \Rightarrow f(x) < f(x')$

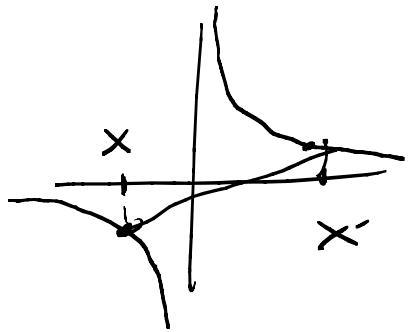
DECRESCENTE

$\forall x, x' \in \text{dom } f \quad x < x' \Rightarrow f(x) > f(x')$

$f(x) = -x$



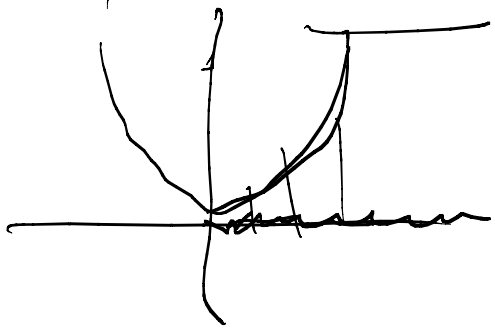
$f(x) = \frac{1}{x}$



$x \neq x' \Rightarrow \begin{matrix} x < x' \\ x > x' \end{matrix}$ oppure f è strett. mon. $\Rightarrow \begin{matrix} f(x) < f(x') \\ f(x) > f(x') \end{matrix}$

$x < x' \not\Rightarrow x^2 < x'^2$

$\forall x, x' \in \mathbb{R}^+_0$



$0 < x'^2 - x^2 =$

$= (x' - x)(x' + x)$

$x' - x > 0$

$x' + x > 0$

$f(x') - f(x)$ ha lo stesso segno di $x' - x$ perché $x' + x > 0$ per $x, x' > 0$.

Risolvere la disequazione $f(x) < f(x')$ e provare che le soluzioni sono le coppie $x < x'$

$x < x'$

$$\frac{f(x) - f(x')}{x - x'} \geq 0$$

f crescente

f strett. crescente $\dfrac{f(x) - f(x')}{x - x'} > 0$

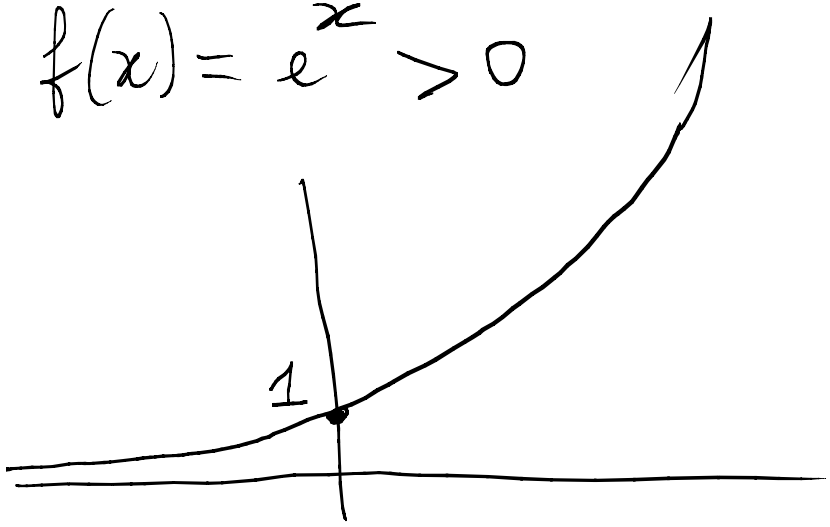
f si dirà MONOTÒNA se è crescente o decrescente

f si dirà STRETT. MONOTÒNA se è strett. cresc. o decresc.

f costante $\quad f : X \to Y$

$\forall\, x, y \in X \quad f(x) = f(y)$

$f(x) = e^x > 0$



$1 + \frac{1}{1\cdot 2} + \frac{1}{1\cdot 2\cdot 3} + \frac{1}{1\cdot 2\cdot 3\cdot 4} + \dots$

$x < y$

$e^x < e^y$

$0 < e^y - e^x = e^x \left(e^{\overbrace{y-x}^{>0}} - 1 \right)$

$>0 \qquad >0$

$f : \mathbb{R} \to \mathbb{R}^+$

è iniettiva e suriettiva $\Rightarrow$ invertibile

$\lg y$ è l'inversa di $y = e^x$

$e^{\lg y} = y \qquad \lg e^x = x$

$f^{-1}(y) =$

$\lg(ab) = \lg a + \lg b \qquad \lg\left(\frac{1}{a}\right) = -\lg a$

$\lg a^b = b \lg a$

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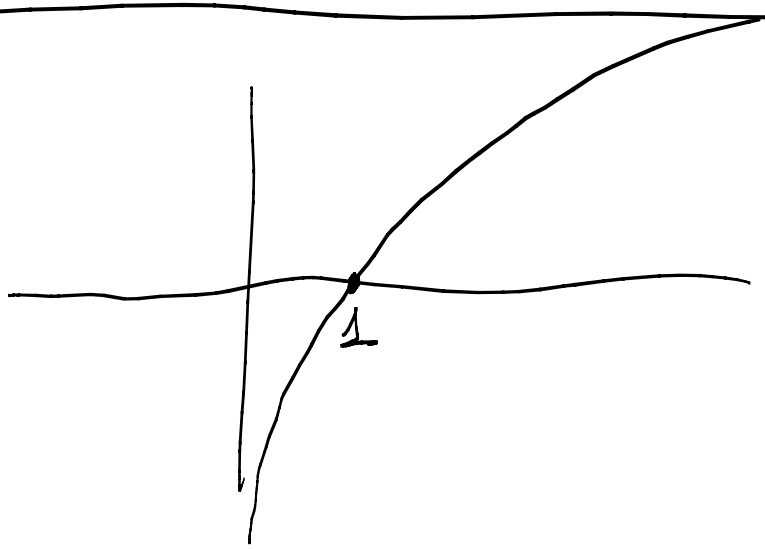
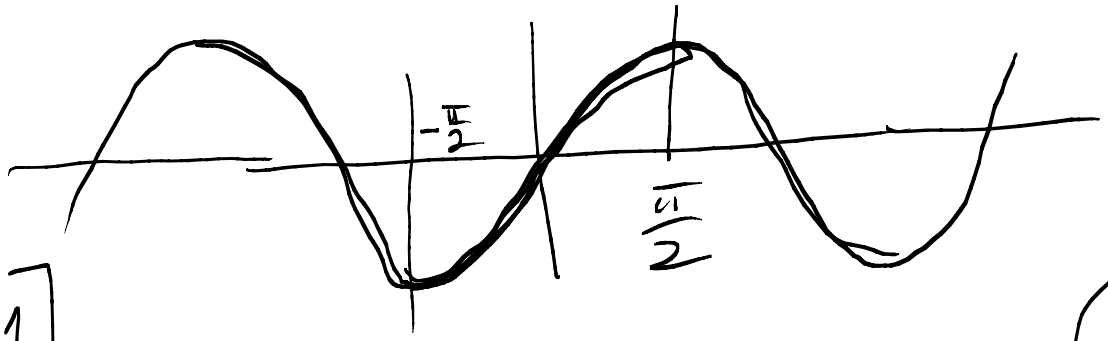
dom $\lg$ è $\mathbb{R}^+$



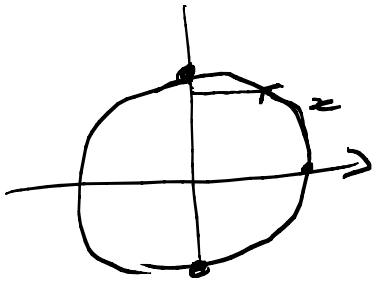
$e^x = 1$

$\underset{\substack{\shortparallel \\ (x)}}{\lg e^x} = \underset{\substack{\shortparallel \\ (0)}}{\lg 1}$

$f(x) = \sin x$



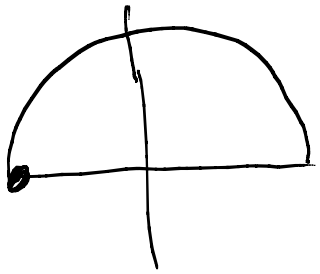
$\sin : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \to [-1, 1]$



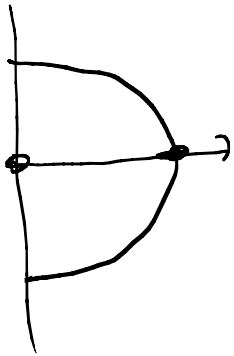
$\arcsin y$ è l'arco in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ per cui il sin vale y

$0 = \arcsin(\sin \pi) \neq \pi$

$\sin(\pi) = 0$



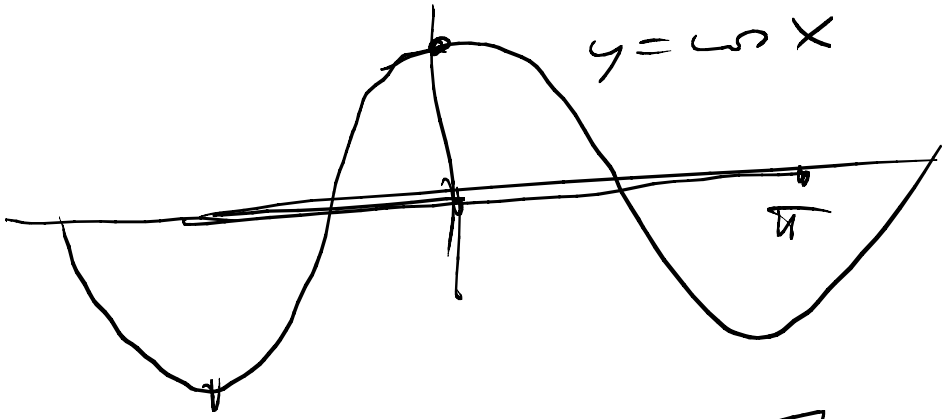
$\arcsin(0) = 0$



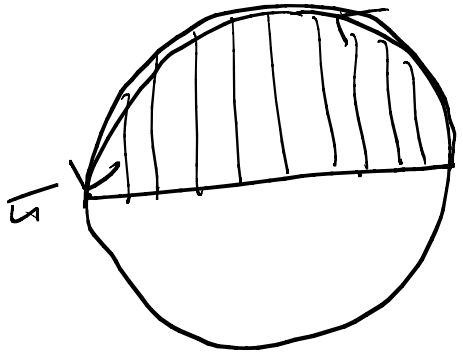
$\arcsin(\sin x) = x$

solo in $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$f(x) = \cos x$



arccos y è l'arco in $[0, \pi]$ in cui il cos vale y.



$\arccos: [-1,1] \to [0,\pi]$

$$x\sqrt{-1-x} = \sqrt{-x^2-x^3}$$

$\forall x$ per cui ha senso

$$\sqrt{x^2(-1-x)} = \sqrt{x^2}\sqrt{(-1-x)} = |x|\sqrt{-1-x}$$

$$\boxed{\sqrt{x^2} = |x|} \quad \forall x \in \mathbb{R}$$

$\sqrt{-1-x}$ definita in numeri

$$x \quad -1-x \geq 0$$

$$\boxed{x \leq -1} < 0$$

$$\sqrt{x^3+2x^2+x} = \sqrt{x(x+1)^2} = \underbrace{|x+1|}_{(x+1)}\sqrt{x} \qquad \underline{\underline{x \geq 0}}$$

$$\sqrt{-x^3-2x^2-x} = \sqrt{-x(x+1)^2} = |x+1|\sqrt{-x} \qquad \boxed{x \leq 0}$$

$$|x+1| = x+1 \quad \text{se} \quad \underline{\underline{x+1 \geq 0}} \quad \underline{\underline{x \geq -1}}$$

$$= -x-1 \quad \text{se} \quad x+1 < 0$$

$$\rightarrow \begin{cases} (x+1)\sqrt{-x} & \text{se} \quad \boxed{-1 \leq x \leq 0} \\ -(x+1)\sqrt{-x} & \text{se} \quad \boxed{x \leq -1} \end{cases}$$

$$\frac{p}{q} \times \frac{p'}{q'} = \frac{pp'}{qq'}$$

$$\frac{p}{q} + \frac{p'}{q'} = \frac{pq' + p'q}{qq'}$$

$$a(b+c) = ab + ac$$

(*) $m = k \cdot p$

$n = k \cdot q$

$r < m$

$\exists\, h, r$:

$m = hn + r$

$\boxed{r < m}$

nelle condizioni (*) ALGORITMO DI EUCLIDE

k è un divisore comune a n ed 2 più

M.C.D

$m = kp$

$n = kq$

$\exists 2 < n \quad \exists h \in \mathbb{N}$

$\boxed{m = hn + 2}$

$kp = hkq + 2 \Rightarrow$

$\Rightarrow 2 = k(p - hq) \Rightarrow$ 2 è divisibile per k.

M.C.D. $\frac{72}{48}$

72	48
48	1
24	

$MCD(72, 48) =$

$= MCD(48, 24) =$

48	24
= 2	

=

* $f(t) \equiv c$ $\quad (f(t) = c \ \forall t \in \text{dom } f)$

$(f)' = 0$

* $\left[f(x) + g(x)\right]' = f'(x) \pm g'(x)$

* $\left[c\, f(x)\right]' = c\, f'(x) \qquad \left[-f(x)\right]' = -f(x)$

* $\left[f(x)\, g(x)\right]' = f'(x)\, g(x) + f(x)\, g'(x)$

* $\left[\frac{1}{f(x)}\right]' = -\frac{f'(x)}{f^2(x)}$

* $\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x)\, g(x) - f(x)\, g'(x)}{g^2(x)}$

$$* \quad \left[g(f(x)) \right]' = g'(f(x)) \cdot f'(x)$$

$$* \quad \left[f^{-1}(y) \right]' = \frac{1}{f'(f^{-1}(y))}$$

$e^{x \ln x} = \boxed{x^x} \quad x > 0$

$$\left[(f(x))^{g(x)} \right]$$

$$f(x)^{g(x)} \equiv e^{\ln f(x)^{g(x)}} =$$

$$= e^{g(x) \ln f(x)}$$