

$\forall N \in \mathbb{N}$

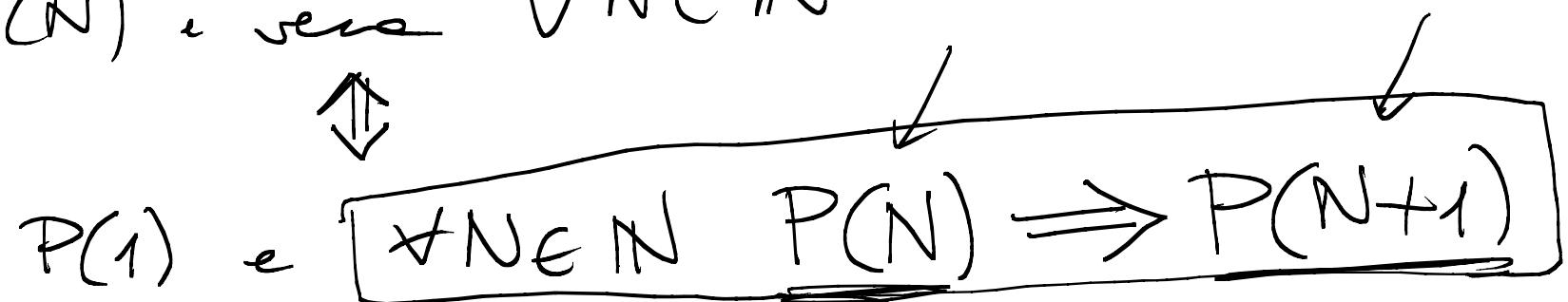
$$\sum_{k=1}^n k^2 = 1^2 + 2^2 + 3^2 + \dots + N^2$$

 \parallel

$$\frac{1}{6}N(N+1)(2N+1)$$

$$\parallel \sum_1^N k^2 = \frac{1}{6}N(N+1)(2N+1) \parallel = P(N)$$

Ts $P(N) \vdash \forall N \in \mathbb{N}$



Ocorre dimostrar che $(P(N+1) \text{ è vero})$

$$\sum_{k=1}^{N+1} k^2 = ? \quad \frac{1}{6} (N+1) (N+1+1) (2(N+1)+1) =$$

$$= \frac{1}{6} \underbrace{(N+1)(N+2)(2N+3)}_{2N^2 + 7N + 6}$$

$$\sum_{1}^{N+1} k^2 = \sum_{1}^N k^2 + (N+1)^2 = \underbrace{\frac{1}{6} N(N+1)(2N+1)}_{\text{caso} \atop \text{corrispondente}} + (N+1)^2$$

d' $P(N)$ che è
sopportato vero

$$= \underbrace{\frac{1}{6}(N+1)}_{\text{caso}} \left[\underbrace{2N^2 + N}_{\text{caso}} + \underbrace{6(N+1)}_{\text{caso}} \right] = \underbrace{\frac{1}{6}(N+1)}_{\text{caso}} (2N^2 + 7N + 6)$$

"TS" $\boxed{1=2}$

"DIM" $a \approx b$

$$\frac{a^2}{a^2} = ab$$

$$a^2 - b^2 = ab - b^2$$

$$(a-b)(a+b) = b(a-b)$$

$$a+b=b$$

$$\cancel{2b}=\cancel{b}$$

$$2=1$$

$$2b=b \quad \underline{b=0}$$



$$2=1$$

$$45=18 \\ 0=0$$

$X \subseteq Y$ X sottinsieme di Y

$$\boxed{\forall x \in X \quad x \in Y}$$

ogni elemento di X

è anche elemento di Y

 $X \subset Y$

$$\left\{ \begin{array}{l} X \subseteq Y \\ Y \not\subseteq X \end{array} \right.$$

$$\forall x \in X \quad x \in Y \quad \text{e} \quad \exists y \in Y \quad y \notin X$$

 $X \supseteq Y$

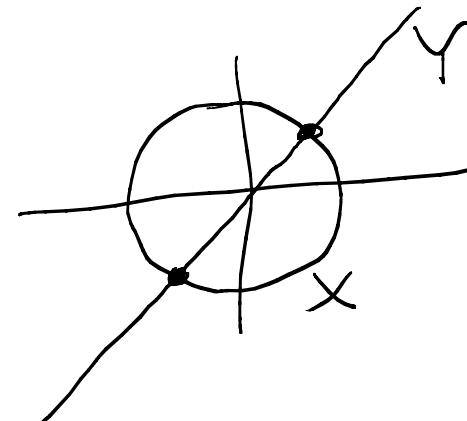
$$X = Y \quad \left\{ \begin{array}{l} X \subseteq Y \\ Y \subseteq X \end{array} \right.$$

 $X \supset Y$

$$X \cap Y = \{x \in X : x \in Y\}$$

el' insiem degli elementi comuni ad X e Y

$$X = \left\{ \underline{(x,y)}^{\in R \times R = R^2} : x^2 + y^2 = 1 \right\}$$



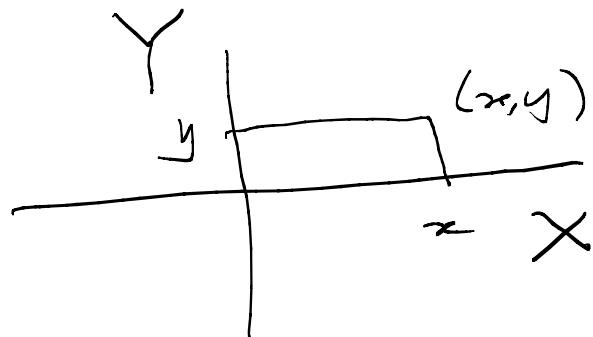
$$Y = \left\{ \underline{(x,y)} : y = x \right\}$$

$$X \cap Y = \left\{ (x,y) : \boxed{\begin{array}{l} x^2 + y^2 = 1 \\ y = x \end{array}} \right\}$$

$$X \cup Y = \{x : x \in X \text{ oppure } x \in Y \text{ oppure ad entrambi}\}$$

X, Y instead

$$X \times Y = \{ (x, y) : x \in X \quad y \in Y \}$$

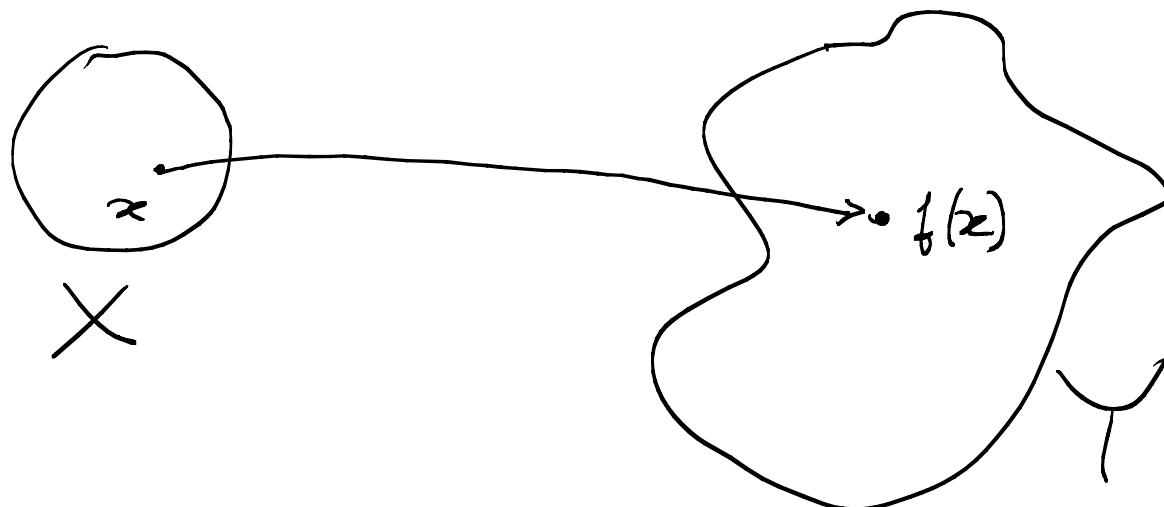


$$\mathbb{R}^2 = \mathbb{R} \times \mathbb{R}$$

$$\mathbb{R}^n = \underbrace{\mathbb{R} \times \mathbb{R} \times \cdots \times \mathbb{R}}_{n \text{ copies}} = \{ (a_1, \dots, a_n) : a_i \in \mathbb{R} \}_{i=1 \dots n}$$

$f : X \rightarrow Y$ X dominio $X = \text{dom } f$
 Y codominio $Y = \text{codom } f$

ad OGNI $x \in X$ associa UN SOLO VALORE $f(x) \in Y$

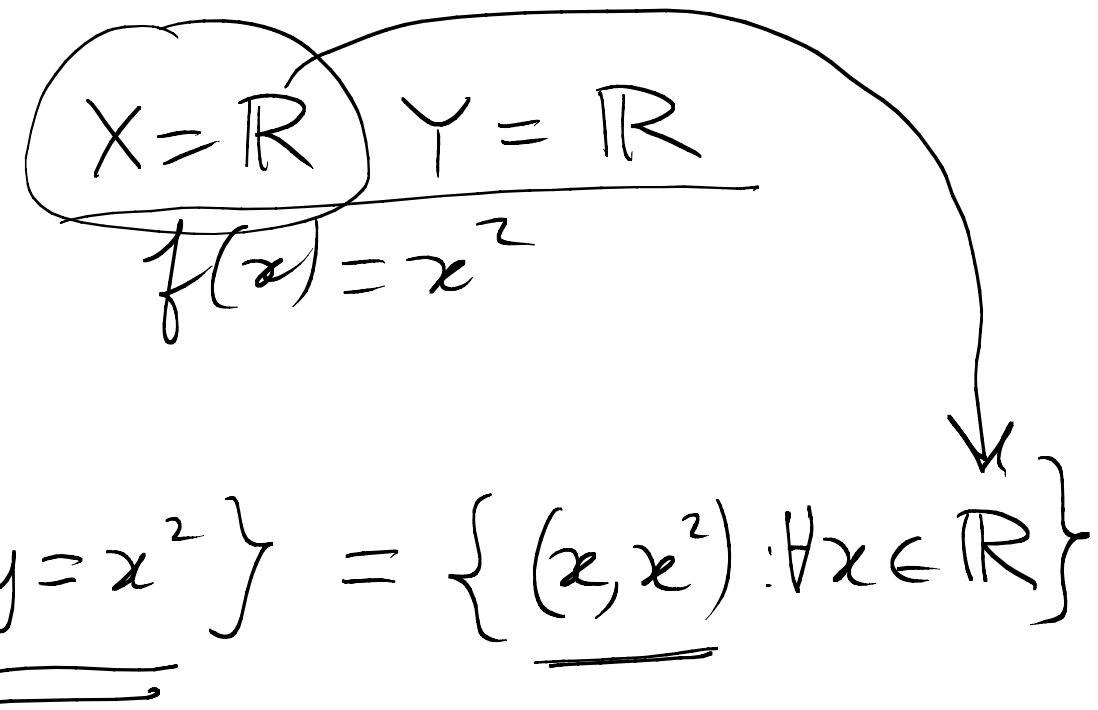
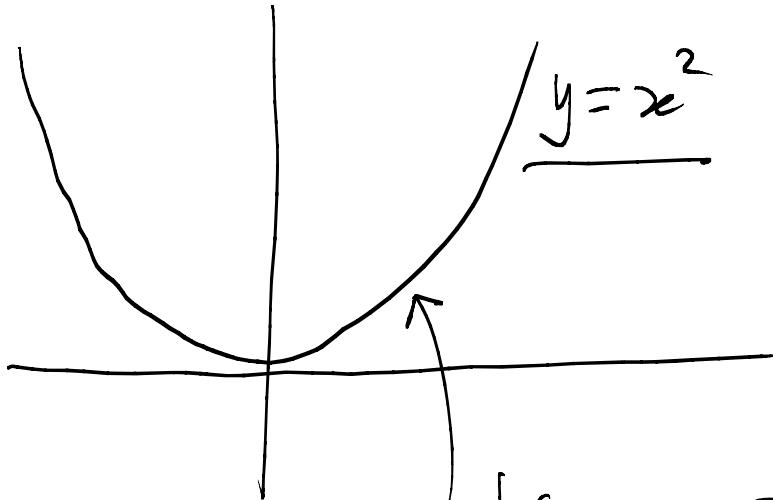


$$X = \mathbb{R} \quad Y = \mathbb{R} \quad f(x) = 0 \quad \forall x \in X$$

$$X = \mathbb{R} \quad Y = \mathbb{R} \quad f(x) = x^2 \rightarrow \text{Im } f = \{y \in \text{codom } f : \exists x \in \text{dom } f \text{ s.t. } y = f(x)\}$$



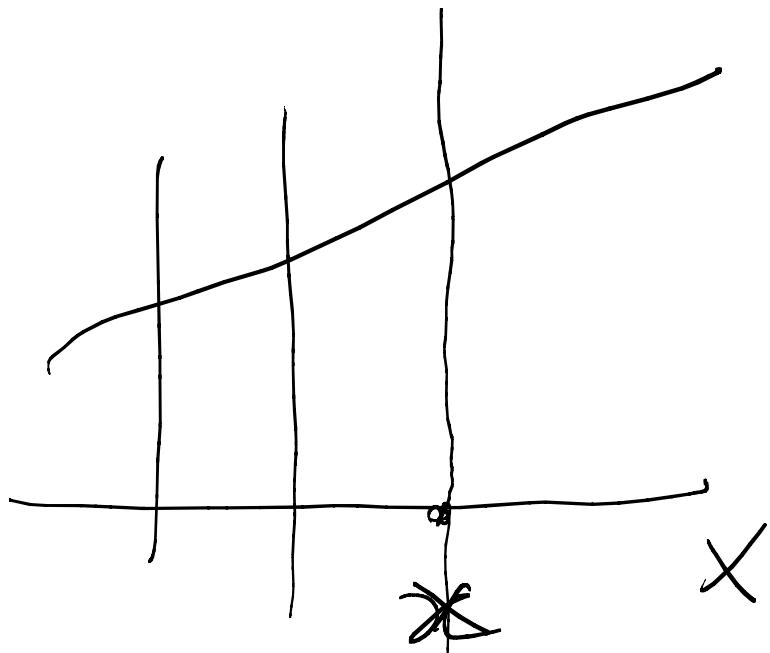
$$f : X \rightarrow Y \quad \text{graph } f = \{ (x, y) \in X \times Y : y = f(x) \}$$



$$\text{Im } f = \{ y \geq 0 \}$$

$$g: \mathbb{Q} \rightarrow \mathbb{Q}$$

$$g(x) = x^2$$



on of the vertex
points for no points
the domain has no points
note

$$f: X \rightarrow Y$$

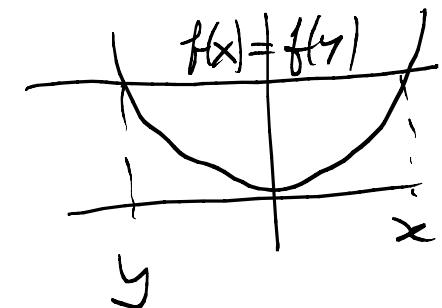
injective se

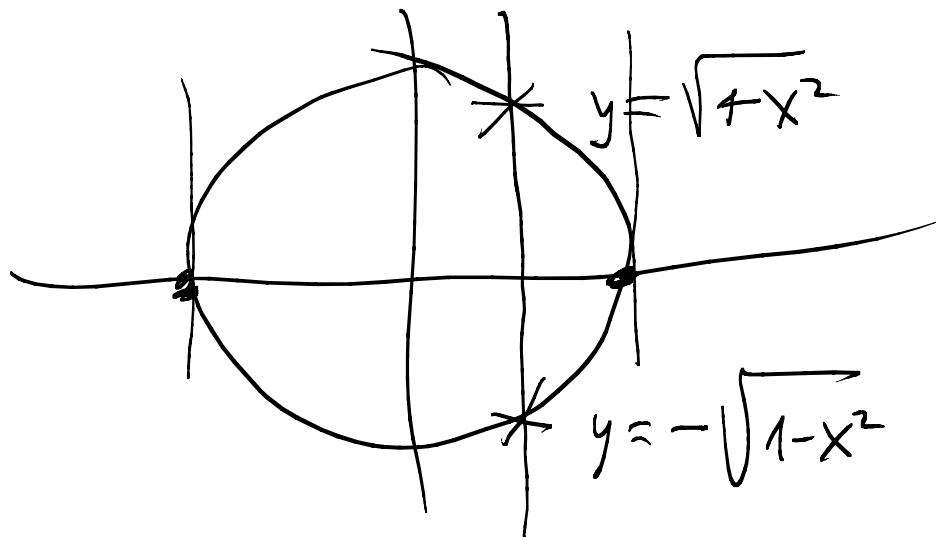
$$f(x) = f(y) \Rightarrow x = y \quad \forall x, y$$

$$x = -1 \quad y = 1$$

$$g(-1) = g(1)$$

$$\begin{array}{c} \cancel{-1=1} \\ -1=1 \end{array}$$





$$[-1, 1]$$

$$\text{dom} = \{1\} \cup \{-1\}$$

$f: X \rightarrow Y$ surjective

$$\forall y \in Y \ \exists x \in X : f(x) = y$$

Invertible $Y = \text{Im } f$

$$y = f(x)$$

quite equation ($x \in \text{dom } f$ $y \in \text{cod } f$)

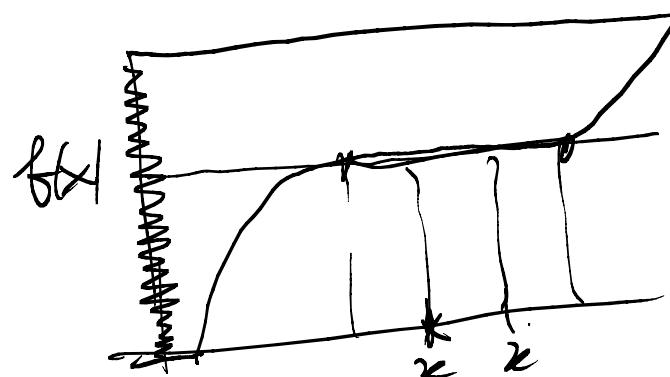
bc
 {

- no solution
- solution w.r.t.

 $x \in \text{dom } f$

bc always one solution for $\exists! y \in \text{cod } f$

f is smooth.



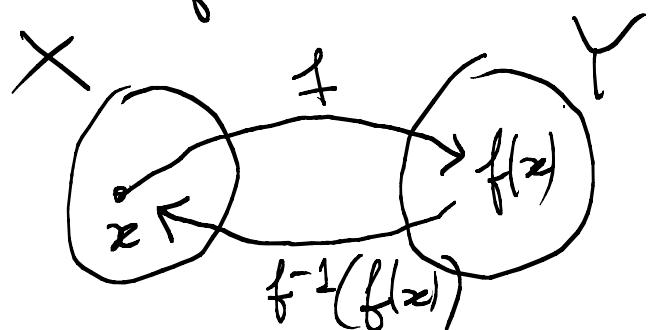
f biiettive se è inversa e snellare

$$f: X \rightarrow Y$$

$y = f(x)$ per una c'è una
soluzione x per ogni
 $y \in Y$

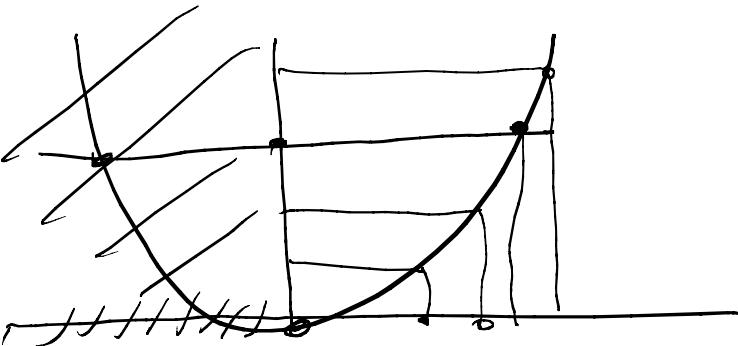
le funzioni che associa a y l'unica soluzione

dell'espressione $y = f(x)$ si dicono funzione
inverse di f e si denota con $f^{-1}(y)$



$$f^{-1}(f(x))$$

$$f(x) = x^2$$



$$\begin{aligned} \text{dom } f &= \mathbb{R}_0^+ = \\ &= \{x \in \mathbb{R} : x \geq 0\} \end{aligned}$$

f è iniettiva da \mathbb{R}_0^+ in \mathbb{R} !

$$x, y \geq 0$$

$$\boxed{x^2 = y^2} \Rightarrow (x-y) \underbrace{(x+y)}_{\geq 0} = 0$$

$f(x) \quad f(y)$

$$x, y > 0$$

$$(x-y) \begin{cases} (x+y) \\ ? \\ > 0 \end{cases} = 0 \Rightarrow \begin{matrix} x-y = 0 \\ \uparrow \\ \text{legge d'annullamento del prodotto} \end{matrix}$$

$$g(t) = t^n \quad n \in \mathbb{N}$$

$n > 1$

$$g : \mathbb{R}_0^+ \rightarrow \mathbb{R}_0^+$$

$f : X \rightarrow Y$

definita su X
a valori in Y

$x, y \geq 0$

$$x^n = y^n \Rightarrow x^n - y^n = 0$$

$$0 = x^n - y^n = (x-y) \underbrace{\left(x^{n-1} + x^{n-2}y + \dots + xy^{n-1} + y^{n-1} \right)}_{> 0} = (x-y) \sum_0^{n-1} x^k y^{n-1-k}$$

annull.
prodotti

$$x = y$$

$$h(t) = t^2 \quad [h: \mathbb{Q}_+^+ \rightarrow \mathbb{Q}_0^+]$$

NON E' SURIETTIVA

$$\boxed{\frac{t}{q}} \left(\frac{t}{q} \right)^2 = \boxed{2 \in \mathbb{Q}_0^+}$$

$$\boxed{t, q \in \mathbb{N}} \quad t^2 = 2q^2 \Rightarrow t^2 \text{ e perf} \Rightarrow t \text{ e perf}$$

$$\Rightarrow t = 2k \text{ e } t^2 = 4k^2 \Rightarrow 4k^2 = 2q^2$$

$$\Rightarrow q^2 = 2k^2 \Rightarrow q^2 \text{ e perf} \Rightarrow q \text{ e perf}$$

$$\sqrt[n]{a} \quad a \geq 0$$

l'unico soluzioni positive (o nulle) dell'equazione

$$x^n = a$$

$$\sqrt[4]{4} = \underline{\underline{2}} \quad (\text{più che})$$

Le soluzioni di $x^2 = 4$ sono

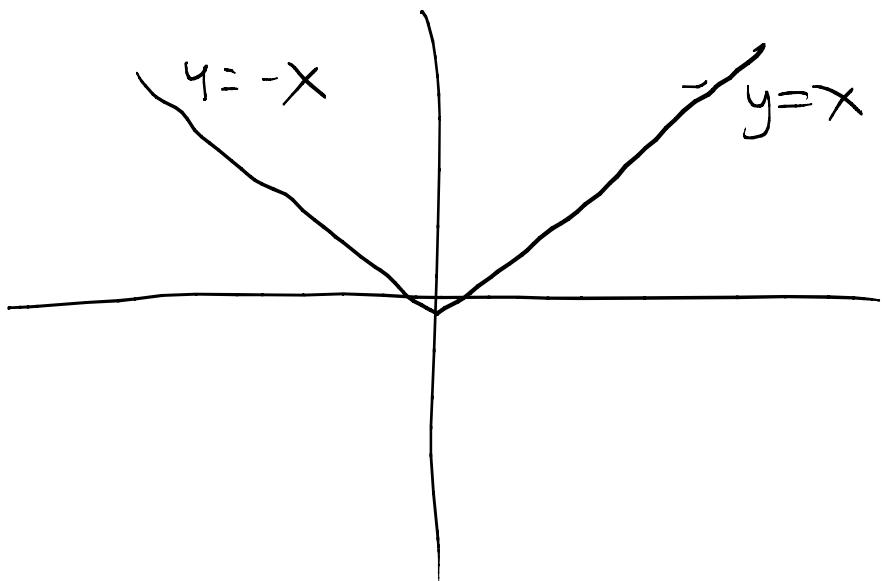
$$\sqrt{4} \text{ e } -\sqrt{4}$$

$$\sqrt{x^2} = x \quad \forall x \in \mathbb{R} \text{ è } \underline{\text{FALSA!}}$$

L. los puntos

$$\sqrt{x^2} = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

~~$\forall x \in X$~~
~~E' VERA~~



$\arcsin(\sin x)$

