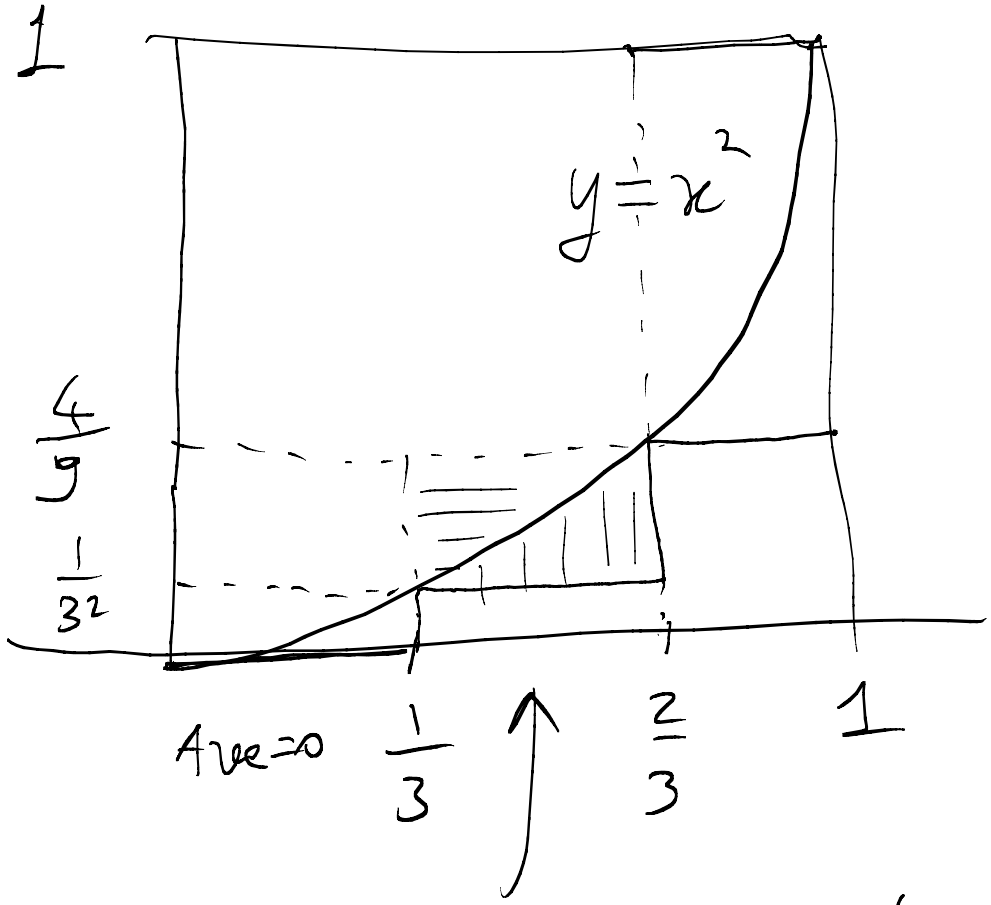
Note Title 9/29/2021



Stima del basso (per difetto)

$$0 + \frac{1}{3}\left(\frac{1}{3^2}\right) + \frac{1}{3}\left(\frac{2}{3}\right)^2$$

Stima dell'alto (per eccesso)

$$\frac{1}{3}\left(\frac{1}{3}\right)^2 + \frac{1}{3}\left(\frac{2}{3}\right)^2 + \frac{1}{3}\left(\frac{3}{3}\right)^2$$

$$\text{area} = \frac{1}{3}\,\frac{1}{9} = \frac{1}{27}\left(\frac{1}{N^3}\right)$$

$$\text{Difetto} = \frac{1}{N}\sum_{k=1}^{N}\left(\frac{k-1}{N}\right)^2 = \frac{1}{N^3}\left[1+2^2+3^2+\cdots+(N-1)^2\right]$$

$$\text{Eccesso} = \frac{1}{N}\sum_{k=1}^{N}\frac{k^2}{N^2} = \frac{1}{N^3}\left[1+2^2+\cdots+N^2\right]$$

$$\sum_{k=1}^{N} k^2 = \frac{1}{6}\left[N(N+1)(2N+1)\right]$$

$$\frac{1}{6N^3}\left[\underbrace{(N-1)}_{(N^2-N)}\,N\,\underbrace{\left(2(N-1)+1\right)}_{(2N-1)}\right] \leq \text{AREA} ? \leq \frac{1}{6N^3}\left(N^2+N\right)(2N+1)$$

$$\frac{1}{6}\,\frac{2N^3-3N^2+N}{N^3} \le \; ? \; \le \frac{1}{6}\,\frac{2N^3+3N^2+N}{N^3}$$

$$\frac{1}{3}-\frac{1}{2N}+\frac{1}{6N^2} \le \; ? \; \le \frac{1}{3}+\underbrace{\frac{1}{2N}+\frac{1}{6N^2}}_{>0}$$

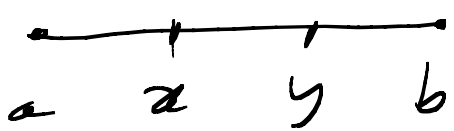
$$< \frac{1}{3} \qquad\qquad < \frac{1}{3}$$

$$\boxed{N>1}$$

$$\underbrace{\frac{1}{3}+\frac{1}{2N}+\frac{1}{6N^2}}_{\text{estremo destro}} - \underbrace{\frac{1}{3}+\frac{1}{2N}-\frac{1}{6N^2}}_{\text{opposto dell'estremo sinistro}} = \text{ampiezza}$$

$$\frac{1}{N}$$

$y > x$



$$\boxed{\begin{array}{l} x \in [a,b] \\ y \in [a,b] \end{array}}$$

$x, y \in [a,b]$

$$y - x \leq \underline{\underline{b-a}}$$

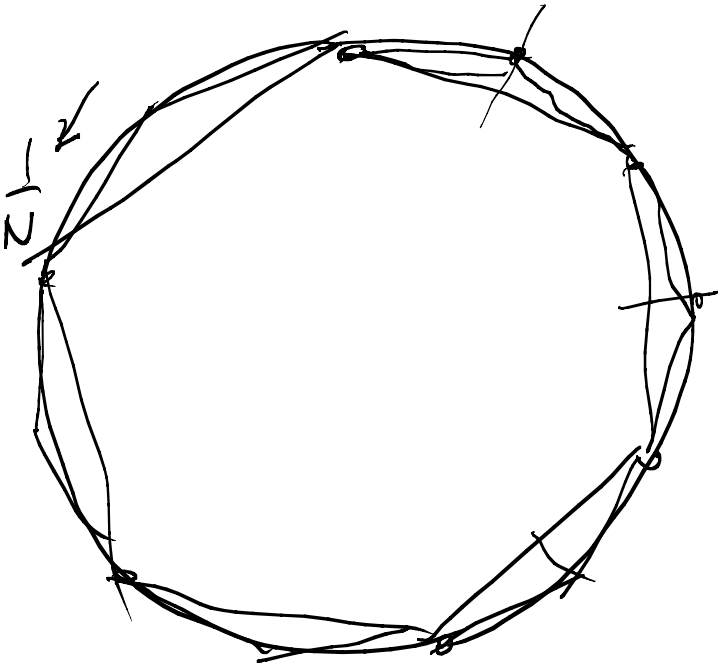
$$\underline{\underline{|x-y|}}$$

x = area richiesta

$y = 1/3$

$$\left|\text{area} - \frac{1}{3}\right| < \frac{1}{N}$$

$\forall N \in \mathbb{R}$



$$\pi \in \left[3 + \frac{10}{71}, 3 + \frac{1}{7}\right]$$

96 lati

$$\underbrace{1+2+\dots+k+\dots+N}_{N \text{ addendi}} = aN^2+bN+c$$

$$\frac{1}{2}N(N+1)$$

$N=1$

$$1 = a+b+c$$

$1+2=$ $$3 = 4a+2b+c$$

$1+2+3=6$ $$= 9a+3b+c$$

vera per $N=1,2,3$

$\forall n \in \mathbb{N}$ ↓ appartiene

$\mathbb{Q}$ frazioni $\frac{p}{q}$, $p,q \in \mathbb{Z}$ numeri razionali

$\mathbb{R}$ reali // $\mathbb{C}$

$\mathbb{N} = 1,2,3,\dots$ (0)

$\mathbb{Z} = 0,+1,-1,+2,-2,\dots$

$\forall n \in \mathbb{N}$ vale la proprietà $P(n)$

$$P(n) = \text{"} \sum_{1}^{n} k = \frac{1}{2} n(n+1) \text{"}$$

$\forall n \in \mathbb{N}$ vale $P(n)$ equivale a

① $P(1)$ è vera

② $P(n) \Rightarrow P(n+1)$ è vera $\forall n \in \mathbb{N}$

② implica ①

$P(1)$ è vera $\xRightarrow{②\ n=1}$ $P(2)$ $\xRightarrow{②\ n=2}$ $P(3) \Rightarrow \cdots\cdots$

Provare per induzione che

$$\sum_{1}^{n} k = \frac{1}{2} n(n+1) \equiv P(n)$$

P(1) $\quad \sum_{1}^{1} k = 1 \qquad n=1 \quad \frac{1}{2} 1(1+1) = 1$ $\quad$ 1 è vera

$P(n)$ $\quad \forall n$

$$\sum_{1}^{n} k = \frac{1}{2} n(n+1) \Longrightarrow \sum_{1}^{n+1} k = \frac{1}{2}(n+1)\left[(n+1)+1\right]$$

$\frac{1}{2}(n+1)(n+2)$

$P(n+1)$

$$\sum_{1}^{n+1} k = (n+1) + \sum_{1}^{n} k = (n+1) + \frac{1}{2} n(n+1) =$$

$\sum_{1}^{n} k = 1$

$$= \frac{1}{2}\left[2(n+1) + n(n+1)\right] = \frac{1}{2}(n+1)\left[n+2\right]$$

$$\begin{cases} 1 = a+b+c \\ 3 = 4a+2b+c \\ 6 = 9a+3b+c \end{cases}$$

$c=0$

$$\begin{cases} 3a+b = 2 \\ 5a+b = 3 \end{cases}$$

$$2a = 1$$

$$a = \frac{1}{2}$$

$$\frac{3}{2} + b = 2 \Rightarrow b = \frac{1}{2}$$

$$\frac{1}{2}N^2 + \frac{1}{2}N$$

$\mathbb{N}$

1) cos'è un numero (elemento di $\mathbb{N}$)

2) $0 \in \mathbb{N}$

3) $S(n)$ successivo di n (definito tra)

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a) 0 è un numero

b) $S(n)$ è un numero

c) 0 non è successivo di alcun numero

d) numeri con uguali successivi sono uguali

e) Σ contiene lo 0 e contiene il successivo di ogni suo elemento allora $\Sigma = \mathbb{N}$