Note Title 7/15/2021

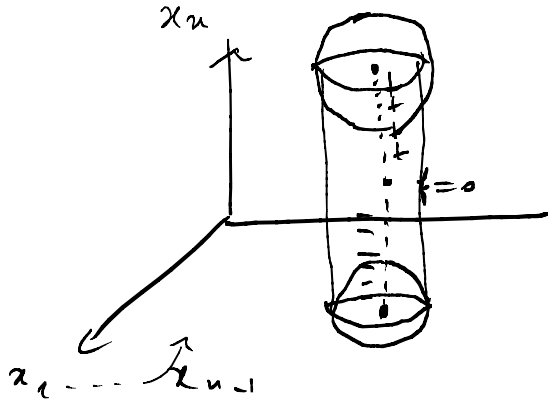
$\lim_n a_n = L \in \mathbb{R}^N$ $\quad \forall \varepsilon > 0 \ \exists \nu : \forall n > \nu \ |a_n - L| < \varepsilon$

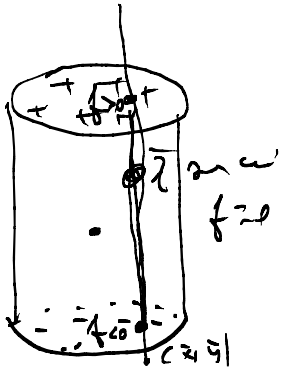
$\Updownarrow$

$a_n \in B_\varepsilon(L)$

$B_\varepsilon(L) = \{ y \in \mathbb{R}^N : |y - L| < \varepsilon \}$

$h(t) = f(\gamma(t))$



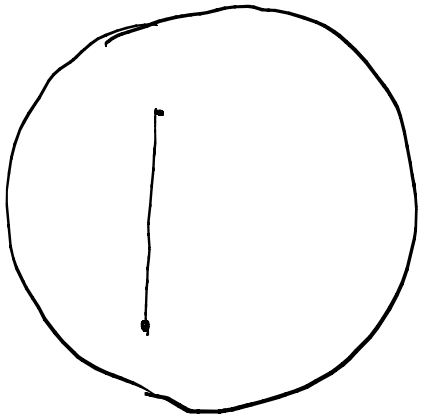


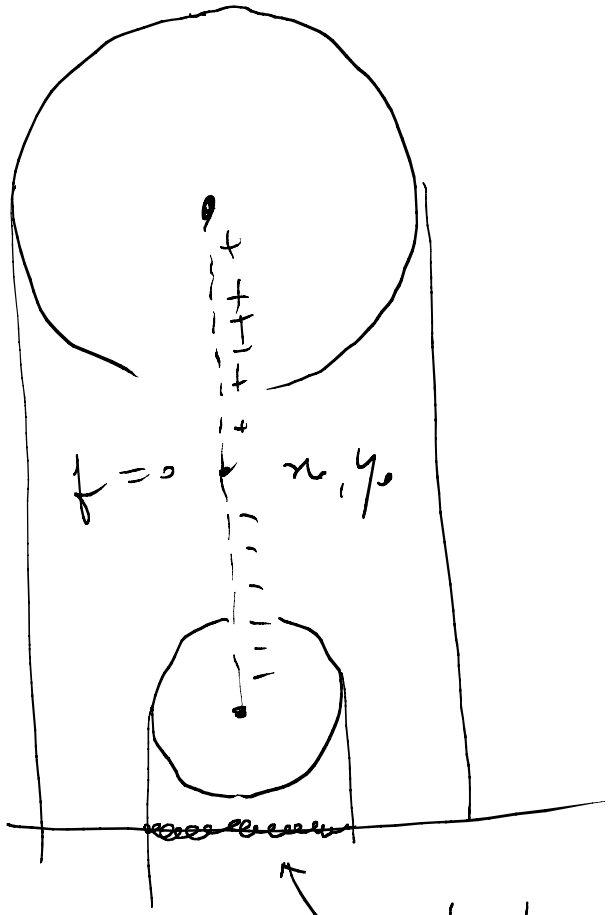
$f(x,y,z) = 0$

$f_z \neq 0$

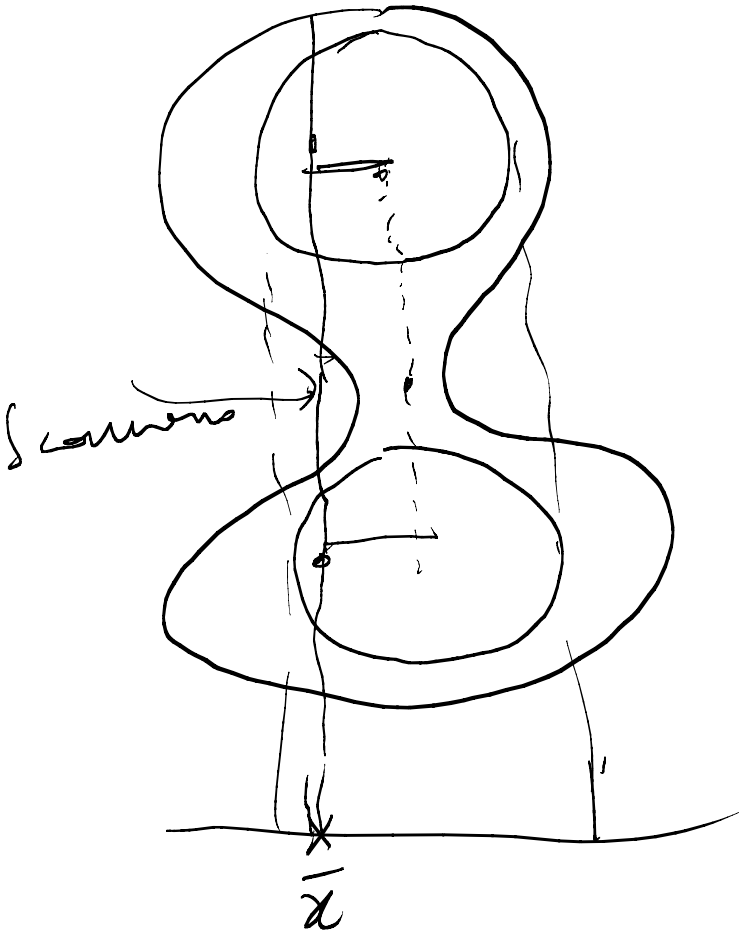
$t \to f(\bar{x},\bar{y},t)$ ← unico zero di

def. $\varphi(\bar{x},\bar{y}) = \bar{z}$





è l'intervallo della tesi $[x_0 - \delta, x_0 + \delta]$



$t \to f(\bar{x}, t)$ è definita su un intervallo

$$p(z) = a_n z^n + a_{n-1} z^{n-1} + \cdots + a_1 z + a_0 =$$

$$\boxed{a_n \neq 0}$$

$$= z^n \left[a_n + \underbrace{\frac{a_{n-1}}{z}}_{\to 0} + \cdots + \frac{a_0}{z^n} \right]$$

$$|p(z)| = \underbrace{|z|^n}_{\to +\infty} \underbrace{\left| a_n + \cdots + \frac{a_0}{z^n} \right|}$$

$$|a_n| > 0 \Rightarrow \left| a_n + \cdots + \frac{a_0}{n} \right| > \frac{|a_n|}{2}$$

$$\boxed{|p(z)| \geq \overbrace{|z|^n \frac{|a_n|}{2}}^{+\infty} \quad \forall z : |z| > \delta} \quad \forall z : |z| > \delta$$

$$f'_{ij} = \frac{\partial f_i}{\partial x_j} \qquad i = 1..M \quad j = 1..N$$

$$\begin{pmatrix} (f_1)_{x_1} & \cdots & (f_1)_{x_n} \\ (f_2)_{x_1} & \cdots & (f_2)_{x_n} \\ \vdots & & \vdots \\ (f_m)_{x_1} & \cdots & (f_m)_{x_n} \end{pmatrix}$$

$f: \mathbb{R}^N \to \mathbb{R}^M$

$$df(x_0, w) = f'(x_0)w$$

vera sempre con il dovuto rispetto delle dimensioni delle operazioni

$f: \mathbb{R} \to \mathbb{R}$ $\quad f'(x_0) \in \mathbb{R}$ $\quad w \in \mathbb{R}$ prodotto di numeri

$f: \mathbb{R}^N \to \mathbb{R}^n$ $\quad f'(x_0) \in \mathbb{R}^{M \times N}$ $\quad w \in \mathbb{R}^n$ prodotto matrice per vettore

$f: \mathbb{R} \to \mathbb{R}^n$ $\quad f'(x_0) \in \mathbb{R}^n$ $\quad w \in \mathbb{R}$ il prodotto è scalare per vettore

$f: \mathbb{R}^n \to \mathbb{R}$ $\quad f'(x_0) \in \mathbb{R}^N$ $\quad w \in \mathbb{R}^N$ il prodotto è quello scalare

$\{ x^3y - x = 1 \} = \Gamma$ $\qquad$ $x^3y - x = k$ $\forall k$

$\begin{cases} f_x = 3x^2y - 1 \\ f_y = x^3 \end{cases}$

$\nabla f = 0$?

$f_y = 0$ se e solo se $x = 0$

$f_x(0, y) = -1 \neq 0$

$(0,0)$ è critico

$\nabla f \neq (0,0)$

$(0,0) \notin \Gamma$

$$\lim_{(0,0)} \frac{1-\cos\sqrt{x^2+3y^2}}{\sqrt{x^2+y^2}} =$$

$$\boxed{\frac{1-\cos t}{t^2} \to \frac{1}{2}}$$

NON E' DEF IN $t=0$

$$= \lim_{(0,0)} \frac{1-\cos\sqrt{x^2+3y^2}}{x^2+3y^2} \cdot \frac{x^2+3y^2}{\sqrt{x^2+y^2}}$$

$t=\sqrt{x^2+3y^2}$

$\to 0$

$$\frac{1}{2}$$

perché è 1-omog:

$\sqrt{x^2+y^2}$ non ha zeri sulla sfera unitaria

$x^2+y^2=1$

$$\lim_{0} \frac{A(w) - B(w)}{|w|} = 0$$

→ 0-omogenea

$$\int_a^b \nabla f(\gamma(t))\, \dot\gamma(t)\, dt$$

$\nabla f(\gamma(t))$: composta di funzioni continue

$\dot\gamma(t)$: C^0 perché $\gamma \in C^1$

$\nabla f \equiv A$ su $\mathbb{R}$

$\Rightarrow \nabla f \in C^0$

1) $\nabla f(\gamma(t))\dot\gamma = \frac{d}{dt}[f(\gamma(t))] \in C^0$

$= \int_a^b \frac{d}{dt}[f(\gamma(t))]\, dt$

$= f(\gamma(b)) - f(\gamma(a))$

$\nabla f \in C^0 \Rightarrow f \in C^1 \Rightarrow f$ differenz.

$\Rightarrow$ si può usare la oss. funz. composta

Poichè $\nabla f = A \in C^0 \Rightarrow f \in C^1 \Rightarrow f$ è differenziabile $\Rightarrow$

$f(\gamma(t))$ è derivabile e $\frac{d}{dt}\left[f(\gamma(t))\right] = \underbrace{\nabla f(\gamma(t))\,\dot{\gamma}(t)}_{\in C^0}$

$$\gamma(t_{i+1}) - \gamma(t_i) = \int_{t_i}^{t_{i+1}} \dot{\gamma}(\tau)\, d\tau$$

$$|\gamma(t_{i+1}) - \gamma(t_i)| = \left| \int_{t_i}^{t_{i+1}} \dot{\gamma}(\tau)\, d\tau \right| \leq \int_{t_i}^{t_{i+1}} |\dot{\gamma}(\tau)|\, d\tau$$

dis. triangolare per integrali ($t_i < t_{i+1}$)

$$\Downarrow$$

$$\sum_{0}^{n-1} |\gamma(t_{i+1}) - \gamma(t_i)| \leq \sum_{0}^{n-1} \int_{t_i}^{t_{i+1}} |\dot{\gamma}(\tau)|\, d\tau$$

$$\int_a^b |\dot{\gamma}(\tau)|\, d\tau$$

$y = f(x)$ $\quad f: \mathbb{R} \to \mathbb{R}$

retta tangente $\quad y - f(x_0) = \underbrace{f'(x_0)(x - x_0)}_{df(x_0,\, x-x_0)}$

$$y = \underbrace{f(x_0) + f'(x_0)(x - x_0)}_{\text{termini di grado 0 e 1 dello sviluppo di Taylor}}$$ esplicita

$\Longrightarrow$

$f: \mathbb{R}^n \to \mathbb{R}$

" $y - f(x_0) = \underbrace{f'(x_0)(x - x_0)}_{\nabla f(x_0)(x - x_0)}$

$$y = f(x_0) + \sum_{1}^{n} f_{x_i}(x_0)\,(x - x_0)_i$$

esplicita

CASI DEI GRAFICI CARTESIANI

$\gamma : \mathbb{R} \to \mathbb{R}^n$ retta tang. nel punto del SOSTEGNO $\gamma(t_0)$

$$\varphi(t) = \gamma(t_0) + t\,\dot\gamma(t_0)$$

$t=0$ nel punto di tangenza

ha lo stesso sostegno di:

$$\psi(t) = \gamma(t_0) + \underbrace{(t-t_0)\,\dot\gamma(t_0)}_{\to\ d\gamma(t_0,\,t-t_0)}$$

$t = t_0$

$\phi : \mathbb{R}^2 \to \mathbb{R}^3$ piano tangente a ϕ in $\phi(u_0,v_0)$

$$\to \psi(\alpha,\beta) = \phi(u_0,v_0) + \phi_\alpha(u_0,v_0)\,\alpha + \phi_\beta(u_0,v_0)\,\beta$$

$$\to \tilde\psi(\alpha,\beta) = \phi(u_0,v_0) + (\alpha-u_0)\,\phi_\alpha(u_0,v_0) + (\beta-v_0)\,\phi_\beta(u_0,v_0)$$

jacobiana di ϕ in u_0,v_0 in cui $(\alpha-u_0)$, $(\beta-v_0)$ sono lineari

$$\vec{v}(u_0, v_0) = \phi_\alpha(u_0, v_0) \times \phi_\beta(u_0, v_0) = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

vettore normale (standard)

$$\phi(u_0, v_0) = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$$

vettore normale al piano

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

componenti dello spostamento sul piano