

$$\lim_{(h, k) \rightarrow (0, 0)}$$

$$\frac{f(x_0 + h, y_0 + k) - f(x_0, y_0) - \nabla f(x_0, y_0) \cdot (h, k)}{\sqrt{h^2 + k^2}} = 0$$

$\textcircled{X}$

$$f(x_0 + h, y_0 + k) - f(x_0, y_0) = \underbrace{f(x_0 + h, y_0 - \epsilon k) - f(x_0 + h, y_0) + f(x_0 + h, y_0) - f(x_0, y_0)}_{\| (h, k) \|} \\ = f_y(x_0 + h, \xi)k + f_x(\eta, y_0)h$$

$$\xi \in [y_0, y_0 + k] \quad f(x_0 + h, y_0 + k) - f(x_0, y_0) - f_x(x_0, y_0)h - f_y(x_0, y_0)k =$$

$$\eta \in [x_0, x_0 + h] \quad = [f_x(\eta, y_0) - f_x(x_0, y_0)]h + [f_y(x_0 + h, \xi) - f_y(x_0, y_0)]k \Rightarrow$$

$$\lim_{(x,y) \rightarrow (0,0)} \frac{\sin(x^2+ty^2)}{\sqrt{x^2+y^2}} = \lim_{t \rightarrow 0} \frac{\sin t^2}{t}$$

$t = \sqrt{x^2+y^2}$   
 $(x,y) \rightarrow (0,0)$   
 $\Rightarrow t \rightarrow 0$

non ε-definite  
in  $t=0$ .

$$\frac{\sin(x^2+ty^2)}{x^2+ty^2} \quad \text{and} \quad \frac{x^2+ty^2}{\sqrt{x^2+ty^2}}$$

1                            0

reite  $\psi$  um

$$\phi(t) = \begin{pmatrix} t \cos t \\ t \sin t \\ t \end{pmatrix}$$

$$t \in [0, 2\pi]$$

$$(-\pi, 0, \pi) \in \phi[0, 2\pi]$$

$$\phi'(t) = \begin{pmatrix} \cos t - t \sin t \\ \sin t + t \cos t \\ 1 \end{pmatrix}$$

Le curve è reg. in ogni punt.

$$\begin{cases} t \cos t = -\pi \\ t \sin t = 0 \\ t = \pi \end{cases}$$

$$\psi(t) = \begin{pmatrix} -\pi \\ 0 \\ \pi \end{pmatrix} + t \begin{pmatrix} -1 & 0 \\ 0 & -1 \\ 1 \end{pmatrix} =$$

$$\phi(\pi)$$

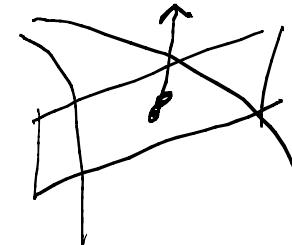
$$\phi'(\pi)$$

$$= (-\pi - t, -\pi t, \pi + t) \equiv \begin{pmatrix} -\pi \\ 0 \\ \pi \end{pmatrix} + t \begin{pmatrix} -1 \\ -\pi \\ 1 \end{pmatrix}$$

punti di tangenza

$$\begin{pmatrix} -\pi \\ 0 \\ \pi \end{pmatrix} + t \begin{pmatrix} -1 \\ -\pi \\ 1 \end{pmatrix}$$

velocità



$$\phi : \Delta^{[a,b] \times [c,d]} \rightarrow \mathbb{R}^3$$

$(x_0, y_0, z_0) = \underline{\phi(u_0, v_0)}$

$$v(u_0, v_0) = \underbrace{\phi(u_0, v_0)}_{\mu} \times \underbrace{\phi(u_0, v_0)}_{\nu} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$\psi(\alpha, \beta) = \underbrace{\phi(u_0, v_0)}_{\text{PARAMETRIC}} + \alpha \underbrace{\phi_u(u_0, v_0)}_{\text{IMPLICIT}} + \beta \underbrace{\phi_v(u_0, v_0)}_{\text{IMPLICIT}}$

$a(x-x_0) + b(y-y_0) + c(z-z_0) = 0$

$(1, 0)$  resp 1

$$y = \tan \frac{\pi}{6} x$$

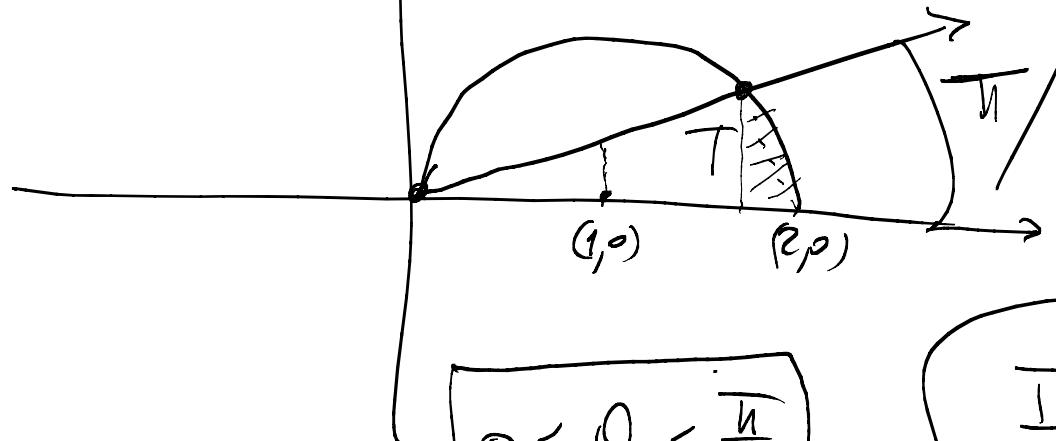
$$\frac{\frac{1}{2}}{\sqrt{3}} = \frac{1}{\sqrt{3}}$$

$$y = \frac{x}{\sqrt{3}}$$

$$\int_1 dy dy$$

T

$$\begin{cases} x > 0; y > 0 \\ (x-1)^2 + y^2 \leq 1 \end{cases}$$



$$0 < \theta < \frac{\pi}{6}$$

$$\begin{aligned} I &\Rightarrow \cos \theta > 0 \\ \overline{II} &\Rightarrow \sin \theta > 0 \end{aligned}$$

$$\theta \in [0, \frac{\pi}{2}]$$

III  $\cos \theta > 0$

$$\int_0^{\frac{\pi}{6}} d\theta \int_0^{2 \cos \theta} r dr$$

$$\begin{aligned} y dx &\text{ oppone } x dy \\ \text{oppone } &\frac{1}{2} [ydx - xdy] \end{aligned}$$

$$|xy| \in (0, \infty)$$

①

$f$  è continua perché  
composte di

$$g(x,y) = xy \quad (\infty)$$

$$\circ t \rightarrow |t| \quad (C^0)$$

1)  $f$  continua? Se no, non è diff.

2) Ha tutte le derivate parziali? Se no,

$$\frac{f(x_0+h, y_0+k) - f(x_0, y_0) - f'_x(x_0, y_0)h - f'_y(x_0, y_0)k}{\sqrt{h^2 + k^2}}$$

$$\lim_{(h,k) \rightarrow (0,0)}$$

esiste ed è nullo?

Fr

as

Non è diff.

→

$$\frac{f(0+h, 0+k) - f(0,0) - 0h - 0k}{\sqrt{h^2 + k^2}} =$$

$$\lim_{(h,k) \rightarrow (0,0)}$$

$$f'_x(0,0) = \lim_{h \rightarrow 0} \frac{f(0+h, 0) - f(0,0)}{h} = \lim_{h \rightarrow 0} \frac{0-0}{h} = 0$$

②

$$f'_y(0,0) = 0$$

$$= \lim_{(0,0)} \frac{|hk|}{\sqrt{h^2 + k^2}} = \underset{\substack{\text{per } h \neq 0 \\ \text{continua in } ||-1||=1}}{0}$$

$$\boxed{f'(x_0)}_{ij} = \frac{\partial f_i(x_0)}{\partial x_j}$$

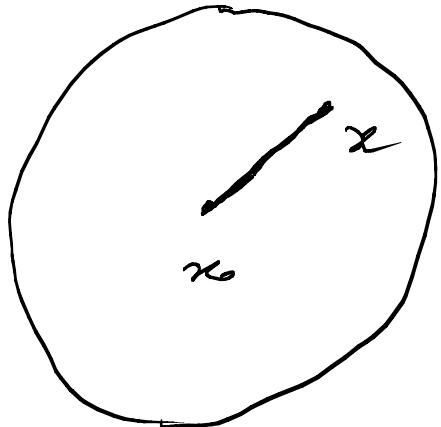
$i=1..n$     $j=1..n$

$$f : \mathbb{R}^n \rightarrow \underline{\mathbb{R}^m}$$

$$f'(x_0) \begin{pmatrix} (f_1^{(x_0)})_{x_1} & (f_1^{(x_0)})_{x_2} & \cdots & (f_1^{(x_0)})_{x_n} \\ \vdots & & & \\ (f_m^{(x_0)})_{x_1} & (f_m^{(x_0)})_{x_2} & \cdots & (f_m^{(x_0)})_{x_n} \end{pmatrix}$$

$m \times n$

$\left\{ \begin{array}{l} x = p \cos \theta \\ y = p \sin \theta \end{array} \right.$     $(p, \theta) \mapsto (x, y)$   
 $T' = \begin{pmatrix} \cos \theta & -p \sin \theta \\ \sin \theta & p \cos \theta \end{pmatrix}$



$$f(x) - f(x_0)$$

$$h(t) = f(x_0 + t(x-x_0)) \quad \underline{t \in [0,1]}$$

$$h(0) = f(x_0) \quad h(1) = f(x)$$

Poiché la funzione è convessa

$$h: [0,1] \rightarrow \mathbb{R}$$

$$h'(t) = \frac{f(x) - f(x_0)}{x - x_0} = \frac{f(x_0 + t(x-x_0)) - f(x_0)}{t(x-x_0)}$$

$\boxed{f(x_0 + t(x-x_0)) - f(x_0)}$

$\overbrace{\phantom{f(x_0 + t(x-x_0)) - f(x_0)}}^{\text{prodotto scalare}}$

$$\frac{f(x) - f(x_0)}{x - x_0} = h(1) - h(0) \xrightarrow{\text{Lagrange}} h'(\xi)(x - x_0) =$$

$$= \nabla f(x_0 + \xi(x - x_0))(x - x_0)$$

$f: \Omega \rightarrow \mathbb{R}$  über die

1)  $f(x_0, y_0) = 0$

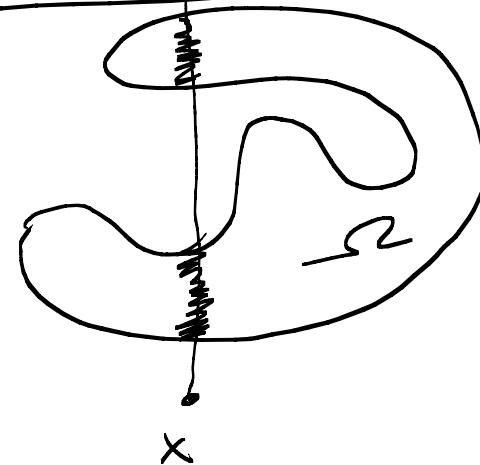
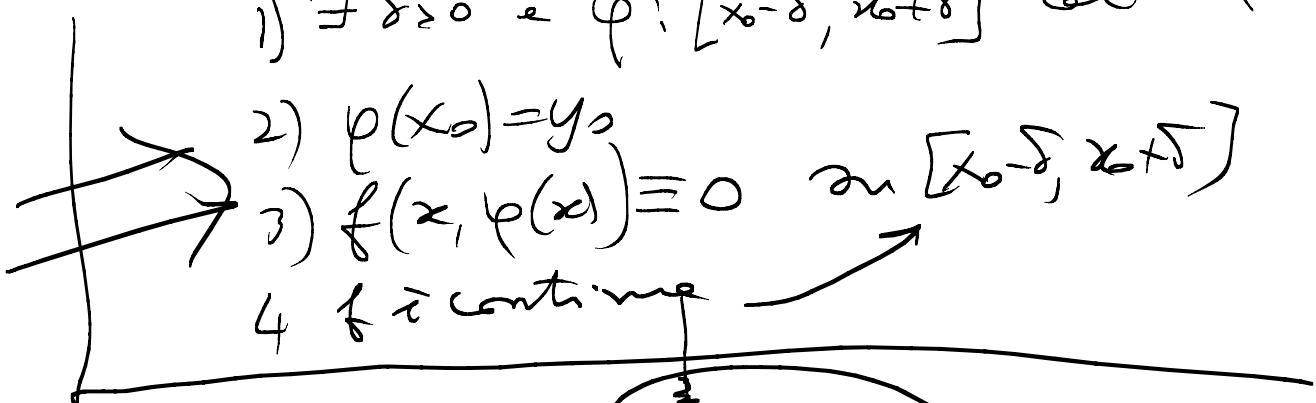
2)  $(x_0, y_0) \in \bar{\Omega}$

3)  $f$  continuous in  $\Omega$

4)  $t \rightarrow f(x, t)$  is stetig

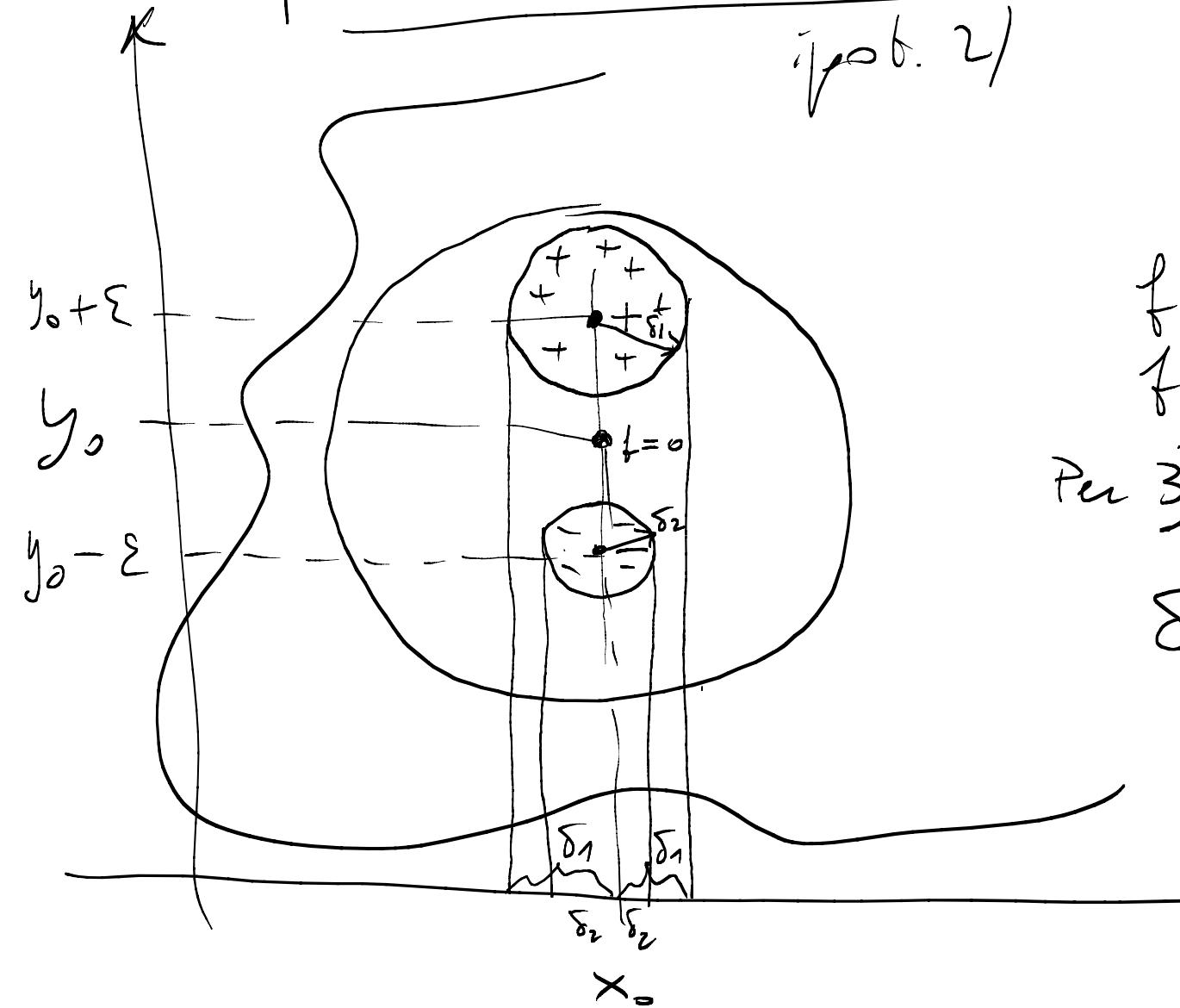
gesucht soll ein  $t^*$  in  $\Omega$  so dass

$$\forall t > t^* \quad f(x, t) > f(x, t^*) \quad \text{so dass } f(x, t^*) \text{ defkt}$$



DIM perché  $(x_0, y_0) \in \bar{\Omega} \quad \exists B_p(x_0, y_0) \subseteq \Omega$

(ipot. 2)



$$f(x_0, y_0 + \varepsilon) > 0 \quad \text{per 4)}$$

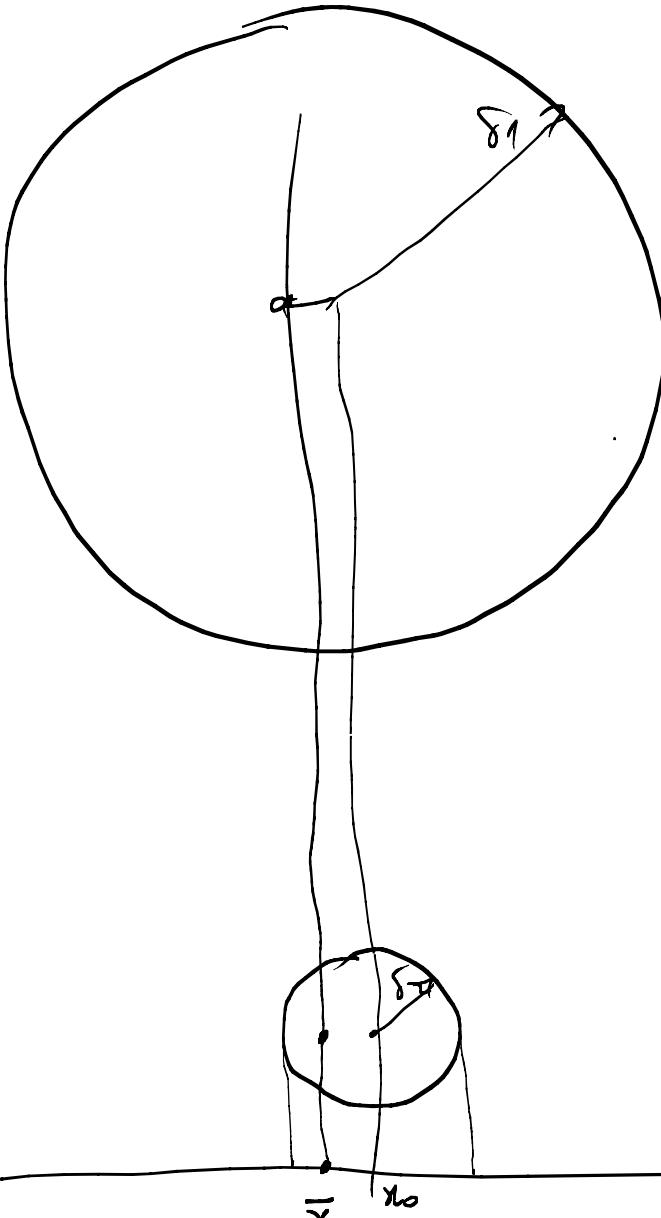
$$f(x_0, y_0 - \varepsilon) < 0 \quad \text{" " "}$$

Per 3) (perm. segno)

$$\delta(\text{delle test}) < \min\{\delta_1, \delta_2\}$$

$[x_0 - \delta, x_0 + \delta] = \text{intestorno}$

delle frontiere di  
due cerchi



$$f(\bar{x}, y_0 + \varepsilon) > 0$$

$$f(\bar{x}, y_0 - \varepsilon) < 0$$

$$\begin{aligned} \text{If } \bar{x} \in [x_0 - \delta, x_0 + \delta] \\ |(\bar{x}, y_0 + \varepsilon) - (x_0, y_0 + \varepsilon)| = \\ = |\bar{x} - x_0| \leq \delta \end{aligned}$$

$t \rightarrow f(\bar{x}, t)$  è continua ed è def. in  $[y - \varepsilon, y_0 + \varepsilon]$  perché i punti  $(\bar{x}, y_0 + \varepsilon)$  e  $(\bar{x}, y_0 - \varepsilon)$

affortuniamo alla fine  $B_p(x_0, y_0)$ , e di conseguenza anche il segmento di estensione  $(\bar{x}, y_0 - \varepsilon) - (\bar{x}, y_0 + \varepsilon)$  appartenente a  $B_p \subseteq \Omega$

$$\text{per il th. degli zeri } \exists \bar{y} : f(\bar{x}, \bar{y}) = 0$$

e si definisce  $\varphi(\bar{x}) = \bar{y}$

$$\rightarrow f(\bar{x}, \varphi(\bar{x})) = 0 \quad \boxed{\begin{matrix} \text{Test} \\ 3 \end{matrix}}$$

il valore  $\bar{y}$  è unico perché la funzione  $t \mapsto f(\bar{x}, t)$  è strettamente crescente.

Li ha mostrato che  $f(x_0, y_0) = 0$  è quindi per l'urto,

$\varphi(x_0) = y_0 \Rightarrow \text{Test 2}$

Teil 4)  $\varphi$  è continua in  $x_0$  fisché

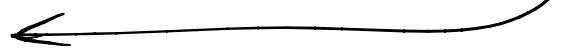
Satto  $\varepsilon > 0$ ,  $\forall x : |x - x_0| < \delta$  ( $\delta$  precedente)

si ha, per estremum  $y_0 - \varepsilon < \varphi(x) < y_0 + \varepsilon$



$$|\varphi(x) - y_0| < \varepsilon$$

$$y_0 = \varphi(x_0) \quad 2)$$



Significato intuito fisché, se i valori  $\varepsilon$  e  $\delta$  visti  
le stesse, le funzioni  $\varphi$  considerate nella precedente  
in tutta la pianta comuni dei loro domini.

$\alpha(x, w)$  (esiste in  $w$  per ogni  $x \in \mathbb{R}^n$ )

è dire ESATTA (o INTEGRABILE) se  $\exists f: \mathbb{R} \rightarrow \mathbb{R}$   
(delta primitiva o potenziale) tale che  $df = \alpha$   
in  $\mathbb{R} \times \mathbb{R}^n$

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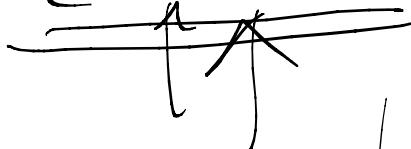
$\alpha(x, w)$  sono detta CHIUSA se il suo campo  
assunto  $A(x)$  (tale che  $\alpha(x, w) = A(x)w$ ) è  
IRROTAZIONALE  $\Leftrightarrow$  è

$$(A_i)_{x_j} = (A_j)_{x_i}$$

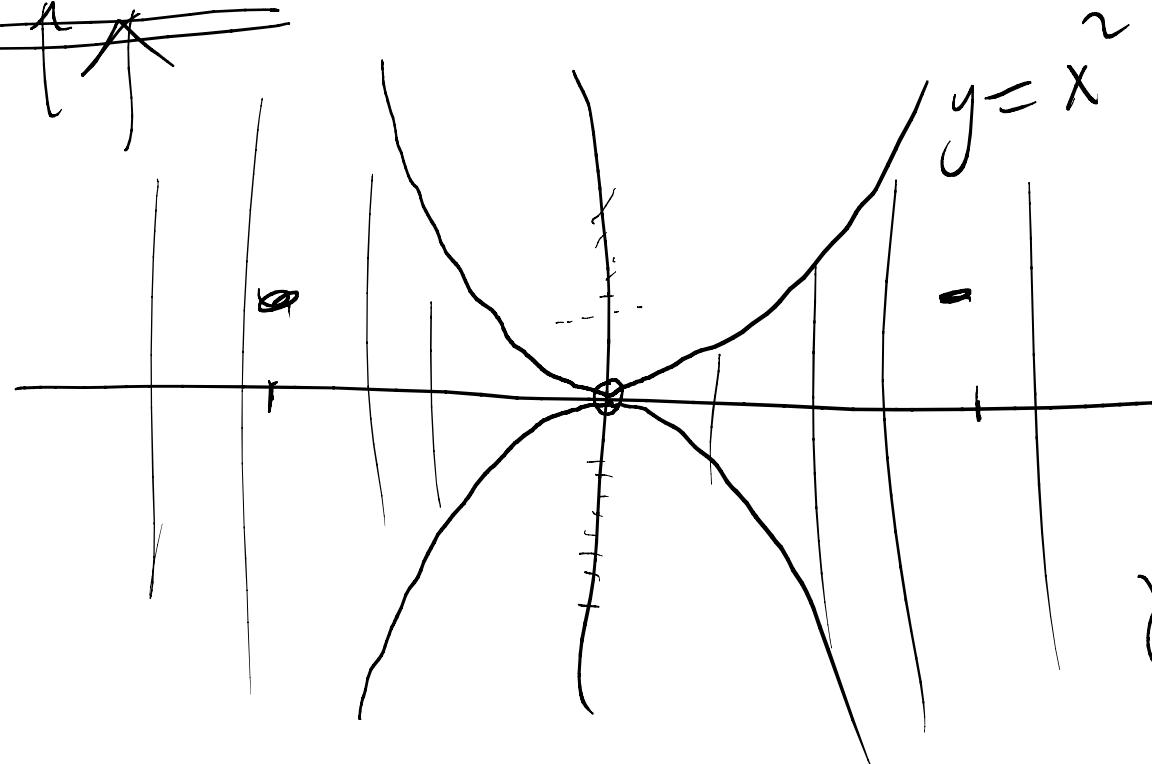
dappertutto in  $\mathbb{R}$

CAMPI E FORME II.

$$\Omega = \{ |y| < x^2 \}$$



$$-x^2 < y < x^2$$

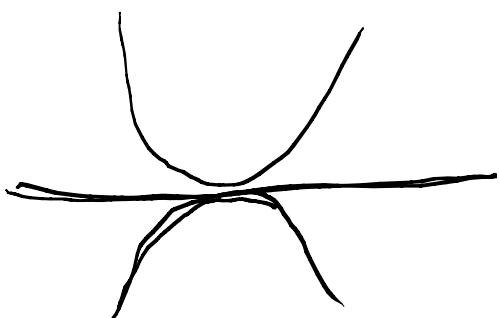


$$(0,0) \notin \Omega$$

Connexità

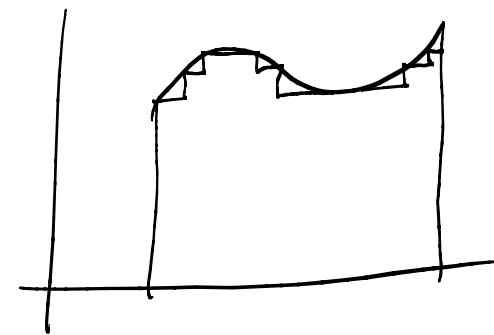
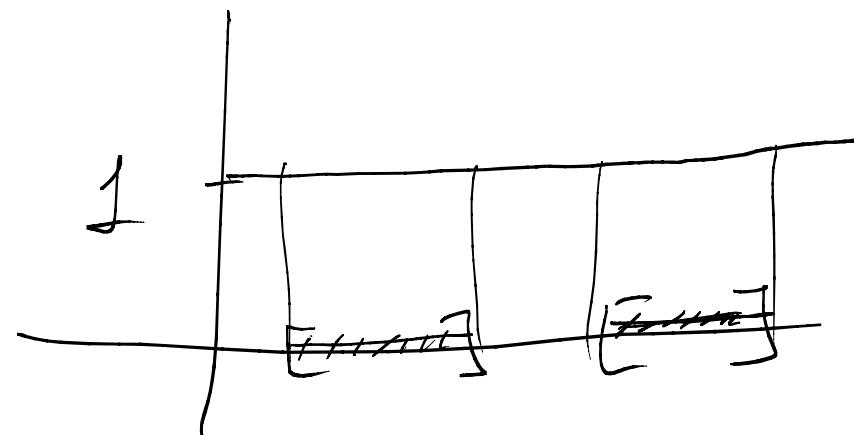
$$\gamma(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix}$$

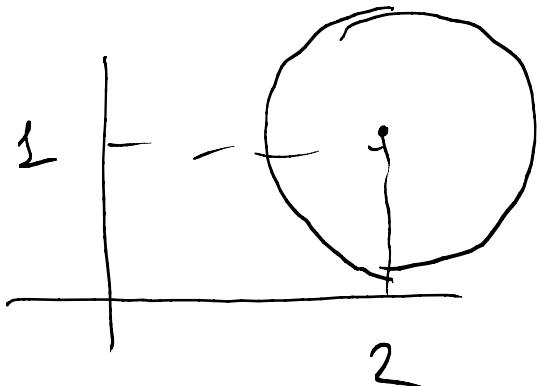
$$\gamma(t)$$



Volume di un cilindro di base  $\Omega$  e altezza 1 =

$$= \text{Aree di } \Omega \times 1$$





$$(x-2)^2 + (y-1)^2 = 1$$

centro  $(2, 1)$  raggio 1

Trovare il punto d'origine  
distanza 2 dall'origine

$$\min x^2 + y^2$$

$$x^2 - 4x + 4 + y^2 - 2y + 1 = 1$$

$$x^2 + y^2 - 4x - 2y + 4 = 0$$

$$g(x, y)$$

$$\frac{f'(x_0)}{f: \mathbb{R}^n \rightarrow \mathbb{R}} \nabla f(x_0) = (f_{x_1}(x_0), \dots, f_{x_n}(x_0))$$

$$f: \mathbb{R}^n \rightarrow \mathbb{R}^m$$

$$f'(x) = \begin{pmatrix} (f_1)_{x_1} & \cdots & (f_1)_{x_m} \\ \vdots & & \vdots \\ (f_m)_{x_1} & \cdots & (f_m)_{x_n} \end{pmatrix} = \begin{pmatrix} \nabla f_1 \\ \nabla f_2 \\ \vdots \\ \nabla f_m \end{pmatrix}$$

$\phi: \Delta \rightarrow \mathbb{R}^3$

f. force. tang.  $\in \phi(u_0, v_0)$

$$\psi(\alpha, \beta) = \phi(u_0, v_0) + \alpha \phi_u(u_0, v_0) + \beta \phi_v(u_0, v_0)$$

é un prem ex-e 26 x  $\phi_u - \phi_v$  sono  
molti ( $\Leftrightarrow \phi_u \times \phi_v \neq 0$ )

$$y = \sqrt{x} \quad x \in [0, 1]$$

$$\gamma(t) = \begin{pmatrix} t \\ \sqrt{t} \end{pmatrix}$$

$$\lambda(\text{graph } y) = \int_0^1 \sqrt{1 + y'^2} \, dx =$$

$$\gamma(t) = \begin{pmatrix} t \\ f(t) \end{pmatrix}$$

$$= \int_0^1 \sqrt{1 + \frac{1}{4x}} \, dx =$$

$$\dot{\gamma}(t) = \begin{pmatrix} 1 \\ f'(t) \end{pmatrix}$$

$$= \frac{1}{2} \int_0^1 \frac{\sqrt{4x+1}}{\sqrt{x}} \, dx \quad \overline{t = \sqrt{x}}$$

$$\int_0^1 \frac{\sqrt{1+4t^2}}{t} 2t \, dt =$$

funktion  
vertauschen

$$du = 2dt$$

$$2t = u \quad du = 2dt$$

$$= \frac{1}{2} \int_0^2 \sqrt{1+u^2} \, du =$$

$$u = \sinh v \quad 2t \, dt = dx$$

$$1 \cdot t = \sqrt{1+4t^2}$$

$$\int \frac{\sqrt{1+u^2}}{u} du = \begin{cases} \frac{\cosh v}{\sinh v} & \cosh v dv \\ \sinh v & \end{cases}$$

$$\begin{aligned} &\sqrt{1+2x} \\ &\sqrt{1+4x} \end{aligned}$$

für  $v$  reelle

$$dv = \frac{2}{\sqrt{1+4x}} dx = \frac{2}{\sqrt{1+4x}} \cdot \frac{1}{2} dx = \frac{1}{\sqrt{1+4x}} dx$$

che  $\approx$  reziproker Funktionswert

$$e^v = s \quad v = \log s$$

$$dv = \frac{1}{s} ds$$

$$\sqrt{1+2x} = t \quad 1+2x = t^2 \quad x = \frac{t^2 - 1}{2} \quad dx = \frac{1}{2} 2t dt$$