

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \phi(u, v)$$

$$\begin{aligned} \phi: \Delta &\rightarrow \mathbb{R}^3 \\ \Delta &\subseteq \mathbb{R}^2 \end{aligned}$$

$$\psi(\alpha, \beta) = \phi(u_0, v_0) + \alpha \phi_u(u_0, v_0) + \beta \phi_v(u_0, v_0)$$

PIANO PARAMETRICO TANGENTE
AL SOSTEGNO DI ϕ in $\phi(u_0, v_0)$

$$\psi: \mathbb{R}^2 \rightarrow \mathbb{R}^3$$

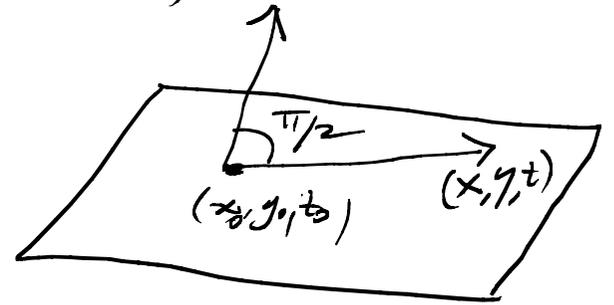
$$\begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \phi(u_0, v_0)$$

$$\phi_u(u_0, v_0) \times \phi_v(u_0, v_0) \neq 0$$

$$\nabla(u_0, v_0) = \phi_u(u_0, v_0) \times \phi_v(u_0, v_0) = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$$

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

$$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} x - x_0 \\ y - y_0 \\ z - z_0 \end{pmatrix} = 0$$



$|x^{2y}|$ in $(0,0)$

$x > 0$

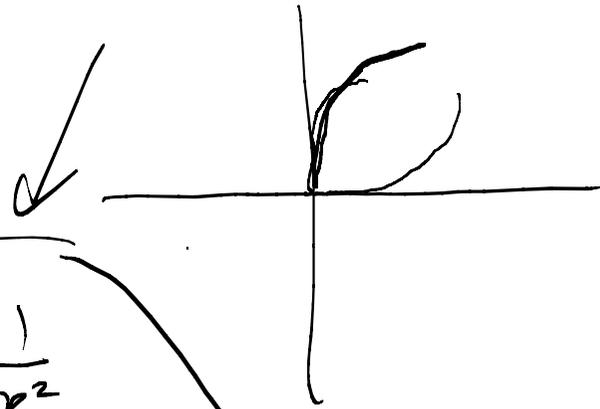
$$x^{2y} = e^{2y \ln x}$$

$x^{2y} \equiv e^{2y \ln x}$
 $x > 0$

$|x|^{2y}$
?

$(x,y) \neq (0,0)$

$\lim_{(0,0)} |x|^{2y}$

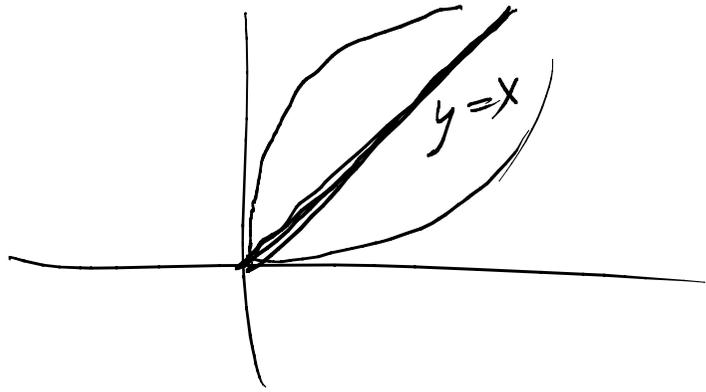


$x > 0$

$f(x) = \begin{cases} e^{-\frac{1}{x^2}} & x \neq 0 \\ 0 & x = 0 \end{cases}$
 $\lim_0 f = 0$

$\lim_0 e^{2t \ln g(t)}$

$t \rightarrow 0 \Rightarrow g(t) \rightarrow 0 \Rightarrow (g(t), t) \rightarrow (0,0)$
 $t \neq 0$
 $e^{2t \ln g(t)} = e^{2t \ln(e^{-1/t^2})} = e^{2t \cdot (-\frac{1}{t^2})} \rightarrow 0$
 $t \rightarrow 0 \Rightarrow -\infty$



$$e^{2y \ln x}$$

$$\lim_{x \rightarrow 0} f(x, x) = 1$$

$$2x \ln x \rightarrow 0$$

$$e^{2x \ln x} \rightarrow e^0 = 1$$

$$(x_1 \dots x_n) \xrightarrow{F} (y_1 \dots y_n)$$

$$\rightarrow \begin{cases} y_1 = F_1(x_1, \dots, x_n) \\ \vdots \\ y_n = F_n(x_1, \dots, x_n) \end{cases}$$

$$F(x_1, \dots, x_n) = \begin{pmatrix} F_1(x_1, \dots, x_n) \\ \vdots \\ F_n(x_1, \dots, x_n) \end{pmatrix}$$

$$\begin{aligned} x_1 &= (F^{-1})_1(y_1, \dots, y_n) \\ &\vdots \\ x_n &= (F^{-1})_n(y_1, \dots, y_n) \end{aligned}$$

0 Lösung
 nicht lösbar
 wegen x_1, \dots, x_n

$$\rightarrow \begin{cases} F_1(x_1, \dots, x_n) - y_1 = 0 \\ \vdots \\ F_n(x_1, \dots, x_n) - y_n = 0 \end{cases} \quad \begin{array}{l} n \text{ Equations} \\ \text{zur Lösung der } x_i \text{ und } y_j \end{array}$$

1) $\in C^1$

2) $(x_0, y_0) : F(x_0) = y_0$

3) $(x_0, y_0) \in \overset{0}{\text{dom}} F$

4) $\det \frac{\partial (G_1 \dots G_n)}{\partial (x_1 \dots x_n)} (x_0, y_0) \neq 0$

$\det \frac{\partial (F_1 \dots F_n)}{\partial (x_1 \dots x_n)} (x_0, y_0) \neq 0$

$\exists \delta > 0 \quad \varphi: B_\delta(x_0) \rightarrow \mathbb{R}^n :$

$G_i(x_1 \dots x_n, y_1 \dots y_n) = F_i(x_1 \dots x_n) - y_i$

$G = \begin{pmatrix} G_1 \\ G_2 \\ \vdots \\ G_n \end{pmatrix}$

$\frac{\partial (G_1 \dots G_n)}{\partial (x_1 \dots x_n)} = \begin{pmatrix} \frac{\partial F_1}{\partial x_1} & \dots & \frac{\partial F_1}{\partial x_n} \\ \vdots & & \vdots \\ \frac{\partial F_n}{\partial x_1} & \dots & \frac{\partial F_n}{\partial x_n} \end{pmatrix} = \frac{\partial (F_1 \dots F_n)}{\partial (x_1 \dots x_n)}$

$G(\varphi(y), y) \equiv 0 \Leftrightarrow F(\varphi(y)) - y = 0$
 $F(\varphi(y)) = y$
F e phi sono inverse l'una dell'altra.

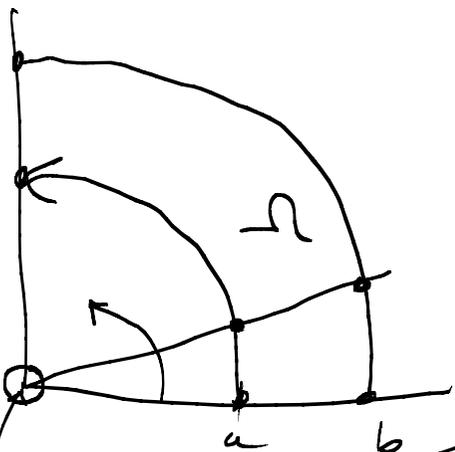
$$\underline{\underline{\varphi'(y) = \left[\frac{\partial(G_1 \dots G_m)}{\partial(x_1 \dots x_n)} \right]^{-1} \cdot \frac{\partial(G_1 \dots G_m)}{\partial(y_1 \dots y_n)}(x, y) \cdot (-1)}}$$

$$\longrightarrow \left[F'(y) \right]^{-1} \begin{bmatrix} -1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{bmatrix} = \underline{\underline{(-1) (-1)^n [F'(y)]}}$$

$$\begin{pmatrix} -1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{pmatrix} = - \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & -1 \end{pmatrix} = \dots = (-1)^n I$$

TEOR. FUNT. IMPLICITE

C^1 2^e
 \longrightarrow disfune



$$F(\rho, \theta) = (x, y)$$

polar param

Jacobians = 0

$$F^{-1}(\Omega)$$

$$= [a, b] \times [0, \frac{\pi}{2}]$$

$$\int_b^h \frac{1}{\sqrt{x^2+y^2}} = \int_0^{\frac{\pi}{2}} d\theta \int_a^b \frac{1}{\rho} \rho d\rho$$

$F^{-1}(\Omega)$

$$\int_{\Omega} f(x) dx = \int_{g^{-1}(\Omega)} f(g(y)) |g'(y)| dy$$

x = g(y) jacobian

Def. $f: \Omega \rightarrow \mathbb{R}$ se dice misurabile se

$f^{-1}([a, b])$ è un insieme per ogni a, b .

Th

Se f è misurabile e limitato su Ω , $|\Omega| < \infty$

allora f è integrabile (con integrale finito \Rightarrow FUBINI)

$$\lim_{x \rightarrow x_0} f(x) = L \quad (1)$$

$$\lim_{y \rightarrow L} g(y) = M \quad (2)$$

$$\lim_{x \rightarrow x_0} g(f(x)) = M$$

x_0 è di accum. per $\text{dom}(f \circ g)$ (3)

Se si aggiunge almeno una delle ipotesi f continua in L o f non definita in L

$\exists \eta$:

$$|g(f(x)) - M| < \varepsilon \text{ è vera se}$$

$$\exists \delta: f(x) \neq L \text{ in } x \in B_\delta(x_0) \setminus \{x_0\}$$

$$\rightarrow |f(x) - L| < \eta \left\{ \begin{array}{l} \text{①} \equiv \forall \eta \exists \delta: |x - x_0| < \delta \wedge x \neq x_0 \wedge x \in \text{dom} f \\ \qquad \qquad \qquad |f(x) - L| < \eta \end{array} \right.$$

$$f(x) \neq L$$

$$\rightarrow f(x) \in \text{dom}(f \circ g) \quad (3)$$

$$y = f(x)$$

$$\text{② } \forall \varepsilon > 0 \exists \eta > 0; y \in \text{dom} g$$

$$\boxed{y \neq L} \quad |y - L| < \eta \Rightarrow |g(y) - M| < \varepsilon$$

$$|g(f(x)) - M| < \varepsilon$$

è vero se

$$|f(x) - L| < \eta$$

$$f(x) \neq L$$

$$f(x) \in \text{dom}(f \circ g) \quad \textcircled{3}$$

$$f(x) = y$$

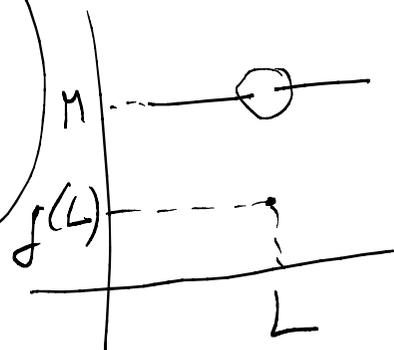
②

$$|g(y) - M| < \varepsilon$$

$$|y - L| < \eta$$

$$y \neq L$$

$$y \in \text{dom } g$$



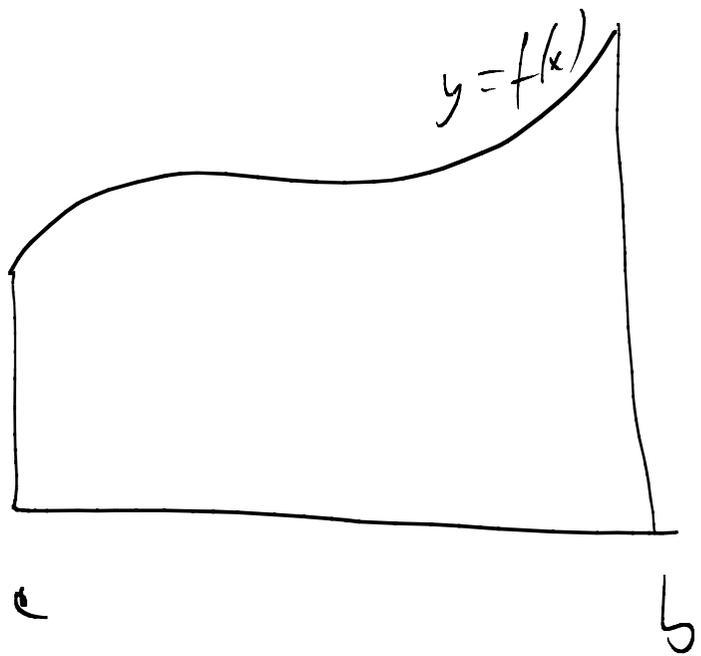
è verificata se $|x - x_0| < \delta$, $x \neq x_0$, $x \in \text{dom } f$

Se g NON è definita in L , la condizione $f(x) \neq L$ non

serve perché $g(f(L))$ non è definita $\Leftrightarrow L \notin \text{dom}(f \circ g)$

e dunque in ogni punto di $\text{dom } f \circ g$ la cond. $f(x) \neq L$ è autom. verificata

- 1) g continua in L
- 2) g non definita in L
- 3) g definita in L e discontinua +
+ $f(x) \neq L$ in una sfera $B_\sigma(x_0)$
escluso al più il punto x_0



$$A(\text{graph } f) = \int_a^b \sqrt{1 + f'(x)^2} dx$$

$f > 0$ area del sottografico $\left\{ (x, y) : \begin{array}{l} x \in [a, b] \\ 0 \leq y \leq f(x) \end{array} \right\}$

$\Rightarrow \int_a^b f(x) dx$

$$f: \mathbb{R}^M \rightarrow \mathbb{R}^N$$

continue

$$\boxed{f \text{ non sia } \equiv 0}$$

$$\lim_{\infty} |f| = +\infty$$



$$|f(x)|$$

ha minimo globale in \mathbb{R}^M .

$$\varepsilon = |f(x_0)| > 0$$

$$\exists \delta: \underline{|x| > \delta} \Rightarrow \underline{|f(x)| > \varepsilon} \Rightarrow |f(x)| > |f(x_0)|$$

osservare che $|x_0| \leq \delta$ perché altrimenti si avrebbe $|x_0| > \delta$
e quindi, per $x = x_0$, $|f(x_0)| > |f(x_0)|$ assurdo!

$$x_0 \in \overline{B_\delta(0)} = \{x \in \mathbb{R}^n : |x-0| \leq \delta\}$$

Per il th. Weierstrass, sia \bar{x} il minimo di $|f(x)|$

su $\overline{B_\delta(0)}$, che è chiuso e limitato.

$$\forall x \in \overline{B_\delta(0)} \quad |f(x)| \geq |f(\bar{x})| \quad \text{perché } \bar{x} \text{ è il punto di minimo.}$$

$$\forall x \notin \overline{B_\delta(0)} \Rightarrow |x| > \delta \Rightarrow |f(x)| > \varepsilon = |f(x_0)| \geq |f(\bar{x})|$$

$x_0 \in \overline{B_\delta(0)}$

$$\Rightarrow \forall x \in \mathbb{R}^n \quad |f(x)| \geq |f(\bar{x})|$$

\uparrow minimo globale in \mathbb{R}^n .

$$\dim(X+Y) + \underset{k}{d(X \cap Y)} = \underset{n}{\dim X} + \underset{m}{\dim Y}$$

X, Y sottosp. di Z

DIM. w_1, \dots, w_k base di $X \cap Y$ ($\dim X \cap Y = k$)

si completa ad una base di X $w_1, \dots, w_k, x_{k+1}, \dots, x_n$ $\dim X = n$

Analogy. per Y $w_1, \dots, w_k, y_{k+1}, \dots, y_m$ $\dim Y = m$

($k < n, k < m, k > 0$)

Il teorema è provato se si verifica che $\dim X+Y = m+n-k$

e ciò è provato se si prova che $w_1, \dots, w_k, x_{k+1}, \dots, x_n, y_{k+1}, \dots, y_m$ è una base per $X+Y$.

$X + Y = \langle w_1, \dots, w_k, x_{k+1}, \dots, x_n, y_{k+1}, \dots, y_m \rangle$
 $\forall z \in X + Y \exists x \in X, y \in Y: z = x + y$

$\forall x \in X \quad \forall y \in Y \quad \exists \alpha_i, \alpha'_i, \beta_j, \gamma_h$ tali che

$$x = \sum_1^k \alpha_i w_i + \sum_{k+1}^n \beta_j x_j$$

$$y = \sum_1^k \alpha'_i w_i + \sum_{k+1}^m \gamma_h y_h$$



$z = x + y \in \langle \underbrace{w_i, x_j, y_h}_{\text{Sistema di generatori}} \rangle$

w_i, x_j, y_h are independent

Let $\alpha_i, \beta_j, \gamma_h$ be

$$\sum_{i=1}^k \alpha_i w_i + \sum_{j=k+1}^n \beta_j x_j + \sum_{h=1}^m \gamma_h y_h = 0$$

$\swarrow \quad \searrow$
 $\underbrace{\hspace{10em}}_x$
 $\underbrace{\hspace{10em}}_y$

$(\Rightarrow \alpha_i, \beta_j, \gamma_h = 0 ?)$

$$w + x + y = 0$$

\Leftrightarrow

$$\underbrace{w+x}_{\in X} = \underbrace{-y}_{\in Y}$$

for $w, x \in X$

$$\Rightarrow \boxed{y \in X \cap Y}$$

$$(w+y) + x = 0$$

$\in \langle w_1, \dots, w_k \rangle$ $\in \langle x_{k+1}, \dots, x_n \rangle$
 " $X \cap Y$

per il lemma di ripartizione della base

le somme $\langle w_1 - w_k \rangle + \langle x_{k+1} - \dots - x_n \rangle = 0$
DIRETTA!

Ne segue che

$$(w+y) + x = 0 \Rightarrow w+y=0$$

$$\in \langle w_1 - w_k \rangle \in \langle x_{k+1} - \dots - x_n \rangle \quad \underbrace{x=0}$$

$$\sum_{i=1}^k \alpha_i w_i + \sum_{h=k+1}^m \gamma_h y_h = 0$$

$$\sum_{j=k+1}^n \beta_j x_j = 0 \Rightarrow \beta_j = 0 \quad \forall j$$

$$\alpha_i, \gamma_h = 0$$

w_i, y_h perché $w_1, \dots, w_k, y_{k+1}, \dots, y_m$ sono
 indip. (base di V)

$$\sum_1^n x_i A_i = B \quad \det \underbrace{(A_1 \ A_2 \ \dots \ A_n)}_A \neq 0$$

$$\Rightarrow x_1 = \frac{1}{\det A} |BA_2 \ \dots \ A_n|$$

$$x_2 = \frac{1}{\det A} |A_1 \ BA_3 \ \dots \ A_n|$$

$$\vdots$$

$$x_n = \frac{1}{\det A} |A_1 \ \dots \ A_{n-1} \ B|$$

Dim

$$|BA_2 \ \dots \ A_n| = \left| \sum_1^n x_i A_i \ A_2 \ \dots \ A_n \right| \stackrel{\text{mult. lin}}{=} \sum_1^n x_i |A_i \ A_2 \ \dots \ A_n|$$

$= x_1 |A_1 \ A_2 \ \dots \ A_n|$

gli altri addendi sono nulli perché \downarrow ha la colonna i -esima ripetuta

