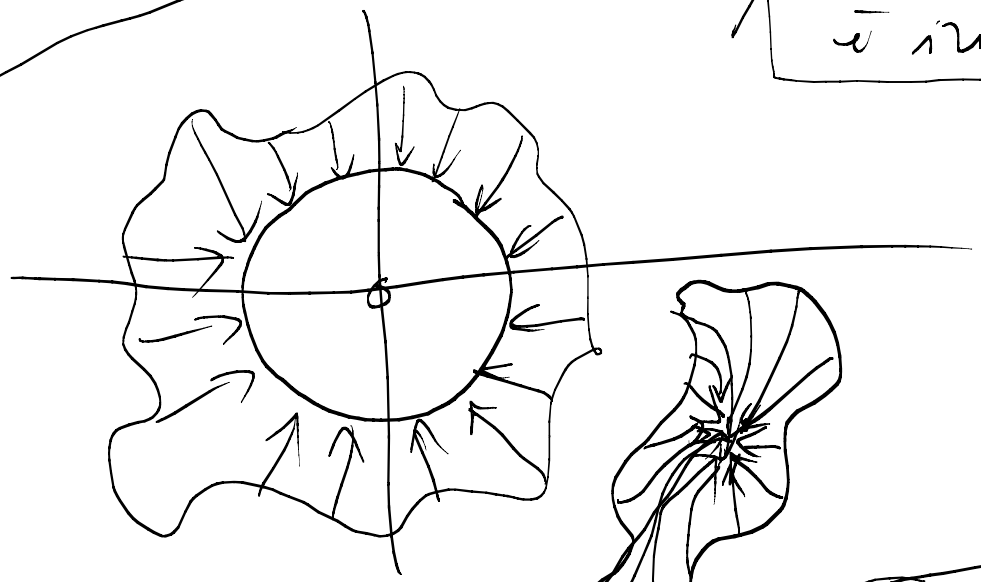
Note Title 5/19/2021

$$A = \nabla\left(\frac{1}{2}\log(x^2+y^2)\right) = \left(\frac{x}{x^2+y^2}, \frac{y}{x^2+y^2}\right) \quad \mathbb{R}^2 \setminus \{(0,0)\}$$

definita su

è irrotaz.

NON È SEMPLIC. CONNESSO

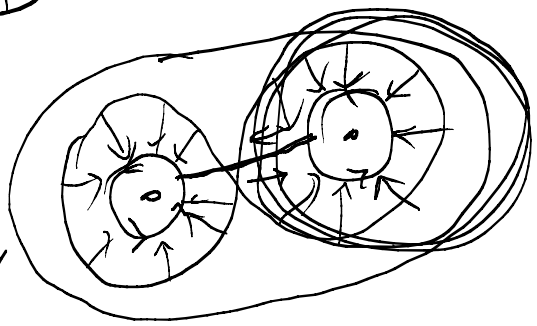


$$\int_\gamma A \quad \text{su } \gamma = \begin{pmatrix}\cos t\\ \sin t\end{pmatrix} \quad t \in [0, 2\pi]$$

$$= 0$$

$$\int A(\gamma(t))\,\dot\gamma(t)\,dt$$

$$\equiv 0 \text{ se } \gamma \text{ è cost.}$$



$$\left(\frac{x}{x^2+2y^2}, \frac{2y}{x^2+2y^2}\right)\begin{pmatrix}\cos t\\ \frac{1}{\sqrt{2}}\sin t\end{pmatrix} = \gamma$$

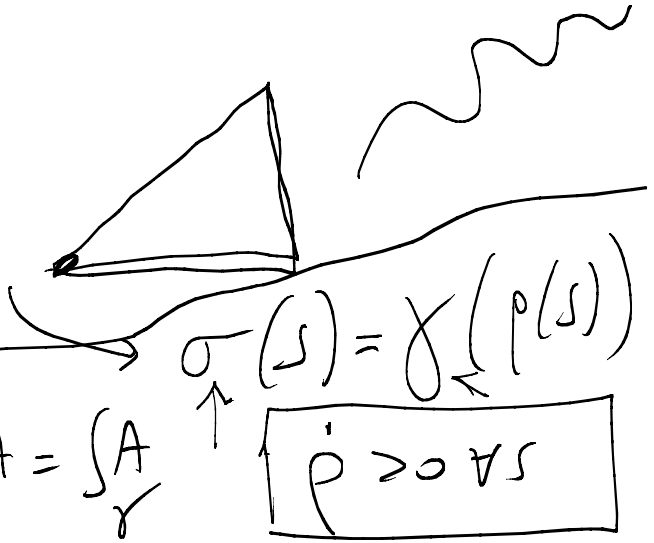
$$\omega(x,y,dx,dy) = x^2 y\,dx + a(x,y)\,dy$$

Determine a in modo che ω sia integrabile (esatta)

imst $\dfrac{\partial}{\partial y}(x^2 y) = x^2 = \dfrac{\partial}{\partial x} a$

$$a(x,y) = \frac{1}{3}x^3 + c(y)$$

$\mathbb{R}^2$



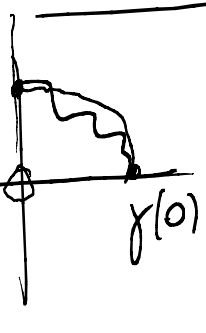
$$\int_\sigma A = \int_\gamma A$$

$$\int_\gamma \left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right)$$

$\sigma(t) = \begin{pmatrix} \cos t \\ \sin t \end{pmatrix}$ sono omotope

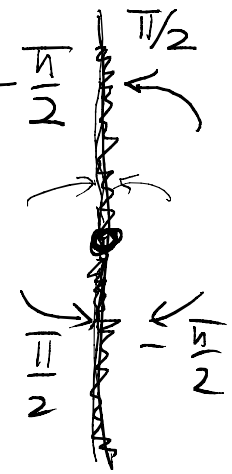
$\gamma(t) = \begin{pmatrix} \cos^3 t \\ \sin^3 t \end{pmatrix}$

I quadrante

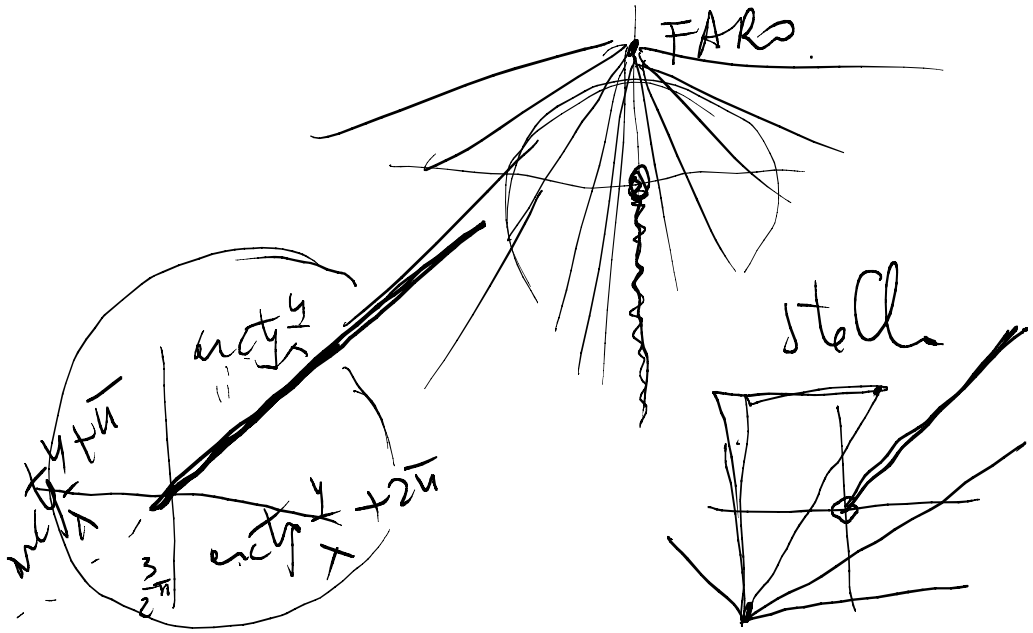


$\arctg \frac{y}{x}$ è una primitiva di $\left(\frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right)$ su $\{x \neq 0\}$

$$\frac{-\frac{y}{x^2}}{1+\frac{y^2}{x^2}} = \frac{-y}{x^2+y^2}$$



$$f(x,y) = \begin{cases} \arctg \frac{y}{x} & x>0 \\ \arctg \frac{y}{x} + \pi & x<0 \\ \frac{\pi}{2} & x=0 \ y>0 \\ -\frac{\pi}{2} & x=0 \ y<0 \end{cases}$$



è una primitiva su $\mathbb{R}^2 \setminus \{(0,y) : y<0\}$

$$f_x = a(x,y) \qquad f(x,y) = \int a(x,y)\,dx + c(y)$$

$$f_y = b(x,y) = \frac{\partial}{\partial y}\int a(x,y)\,dx + c'(y)$$

$x_{\langle u_1 \dots u_n \rangle} =$ $u_1 - u_n$ vettori arbitrari

$= \sum_1^n \alpha_i u_i$ ← proiezione

$$\left(x - \sum_1^n \alpha_i u_i\right) u_j = 0 \qquad \forall j = 1 \dots n$$

n equazioni – n incognite $\alpha_1 - \alpha_n$

$$\begin{cases} x u_1 = \sum_1^n \alpha_i u_i u_1 \\ x u_2 = \sum_1^n \alpha_i u_i u_2 \\ \vdots \\ x u_n = \sum_1^n \alpha_i u_i u_n \end{cases}$$

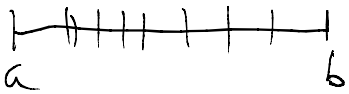
→ incognite

$X + Y =$ X, Y subsets of Z

$$= \{ w \in Z : \exists x \in X, y \in Y \quad x + y = w \}$$

$X + Y = X \oplus Y \iff$ $x + y = 0$, $x \in X$, $y \in Y$ $\Longrightarrow$ $x = y = 0$

$\int_{[a,b]}$ $\longrightarrow$ $\int_a^b f = \begin{cases} \int_{[a,b]} f & a < b \\ -\int_{[a,b]} f & a > b \end{cases}$



$$\int_a^b f\big(\gamma(\rho(s))\big)\,\big|\dot\gamma(\rho(s))\,\dot\rho(s)\big|\,ds = -\int_a^b f(\gamma(\rho(s)))\,|\dot\gamma(\rho(s))|\,\dot\rho(s)\,ds =$$

σ $\qquad$ $\dot\sigma$

$t = \rho(s)$

$$= -\int_{\rho(a)=d}^{\rho(b)=c} f(\gamma(t))\,|\dot\gamma(t)|\,dt = \int_c^d f(\gamma(t))\,|\dot\gamma(t)|\,dt$$

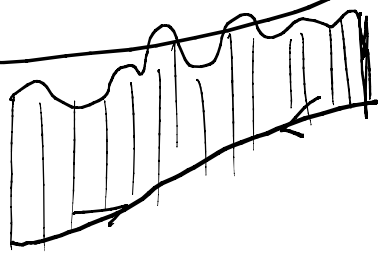
$\rho : [c,d] \to [a,b]$

$\dot\rho < 0$

$\rho(c) = b$

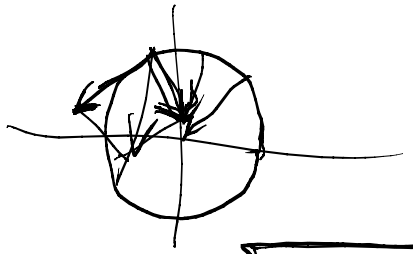
$\rho(d) = a$

estiro



$\gamma(t)$ $\quad \dot\gamma(t) \neq 0$

$$\gamma(t) = \begin{pmatrix} R\cos\omega t \\ R\sin\omega t \end{pmatrix}$$



$$\begin{pmatrix} R\cos\alpha t \\ R\sin\alpha t \end{pmatrix}$$

α ↑ velocità angolare

$$|\dot\gamma| = k \Rightarrow \dot\gamma\ddot\gamma = 0$$

$$\boxed{|\dot\gamma| = \text{cost}}$$

$$\Downarrow$$

$$|\dot\gamma|^2 = k^2$$

$$0 = \frac{d}{dt}\left(\dot\gamma\,\dot\gamma\right) = 2\dot\gamma\ddot\gamma \Rightarrow \boxed{\dot\gamma\ddot\gamma = 0}$$

$$\int_\gamma f \, d\ell \equiv \int_a^b f(\gamma(t)) \, |\dot\gamma(t)| \, dt$$

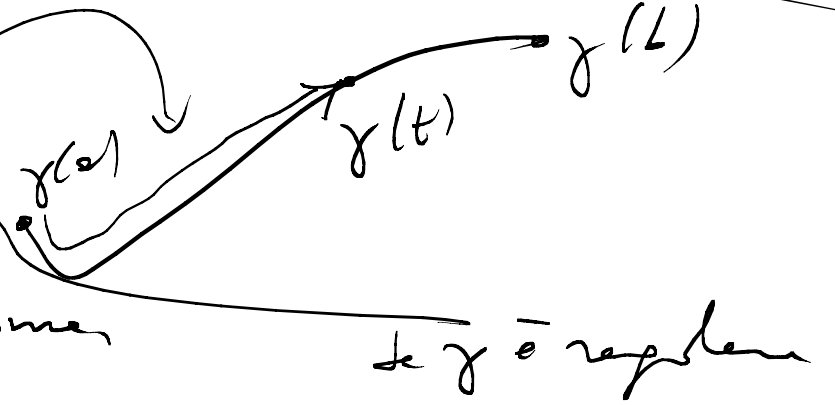
$\gamma : [a,b] \to \mathrm{dom} f$

INVARIANZA

se σ e γ sono equiv.

$$\int_\gamma f = \int_\sigma f$$

1° $\quad s(t) = \int_a^t |\dot\gamma(\tau)| \, d\tau$



verifico $s'(t) = |\dot\gamma(t)| \geq 0$

per Torricelli e $|\dot\gamma(t)|$ continua

se γ è regolare

2° $t \to s(t)$ è continua e strett. crescente da $[a,b] \to [0, L(\gamma)]$

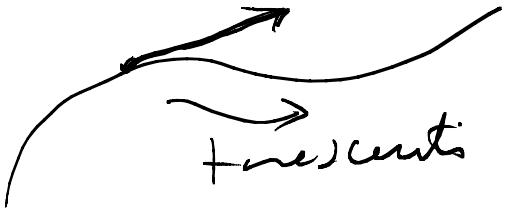
e quindi è invertibile. $\quad t(s)$ è l'inversa

Def $\sigma(s) = \gamma(t(s)) \qquad s \in [0, \Lambda(\gamma)]$

PARAMETRIZZAZIONE INTRINSECA

$$\dot\sigma(s) = \dot\gamma(t(s))\,\underbrace{\dot t(s)}_{\frac{1}{\dot s(t(s))} = \frac{1}{|\dot\gamma(t(s))|}} = \frac{\dot\gamma(t(s))}{|\dot\gamma(t(s))|}$$

versore tangente standard

$|\dot\sigma| \equiv 1 \Rightarrow \boxed{\dot\sigma \cdot \ddot\sigma = 0}$

la velocità $\dot\sigma$ è ortogonale all'accelerazione $\ddot\sigma$ è normale al moto.