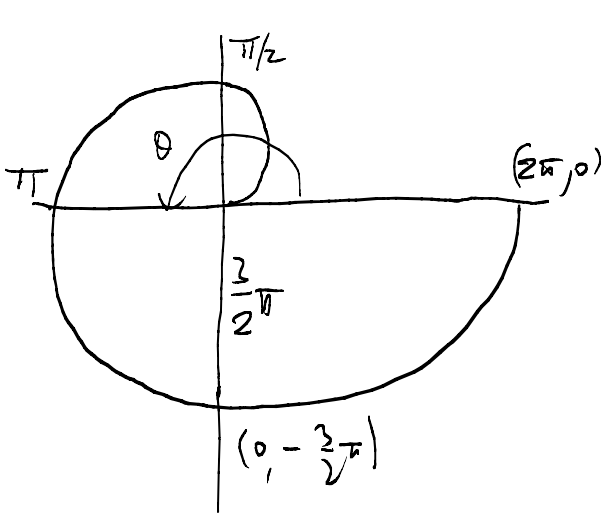
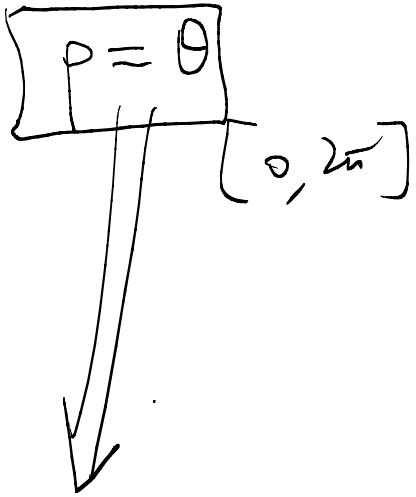
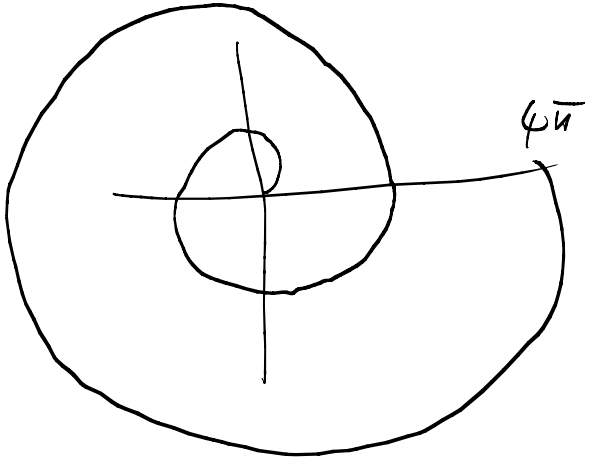
Note Title 5/18/2021







$$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases} \qquad \begin{cases} x = \theta\cos\theta \\ y = \theta\sin\theta \end{cases} \qquad \theta \in [0, 2\pi]$$

$$\frac{dx}{d\theta} = \cos\theta - \theta\sin\theta$$
$$\frac{dy}{d\theta} = \sin\theta + \theta\cos\theta$$

$$\dot{x}^2 + \dot{y}^2 = \underbrace{\cos^2\theta + \sin^2\theta}_{1} + \theta^2\sin^2\theta + \theta^2\cos^2\theta - 2\theta\sin\theta\cos\theta + 2\theta\sin\theta\cos\theta =$$

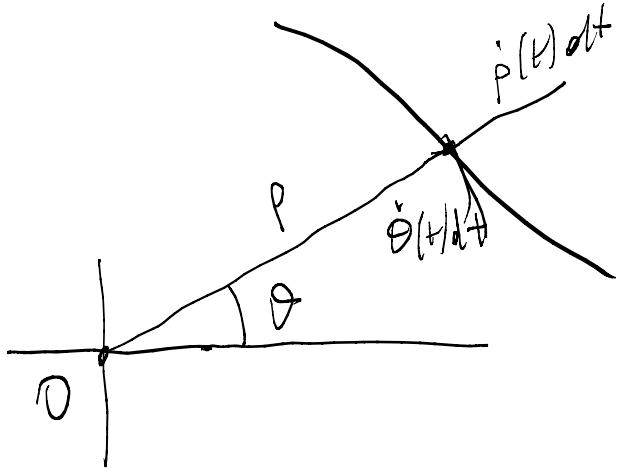
$$= 1 + \theta^2$$

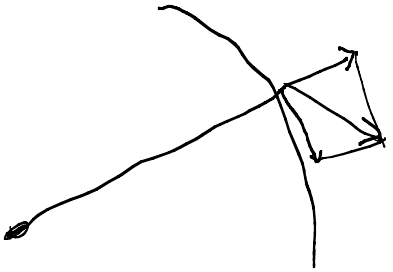
$$|\dot{\gamma}(\theta)| \qquad \Longrightarrow \qquad \Lambda(\gamma) = \int_0^{2\pi} \sqrt{1+\theta^2}\, d\theta$$

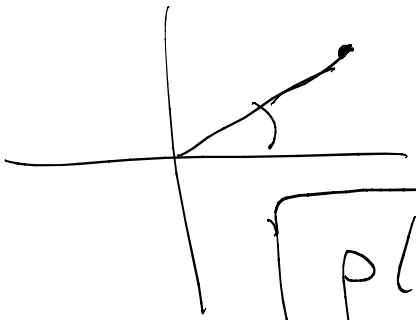
difficile AN 1

FUNZIONI IPERBOLICHE

$\theta = \sinh u$
$d\theta = \cosh u\, du$



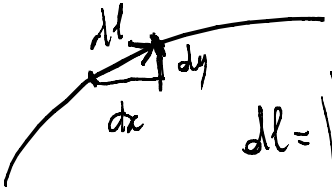




$\rho(\theta)$

RADAR

$\rho(t)\,,\ \theta(t) \qquad t \in [a,b]$



$dl = \sqrt{dx^2 + dy^2}$

$$\Lambda(\gamma) = \int_a^b \sqrt{\dot\rho^2(t) + \rho^2(t)\,\dot\theta^2(t)}\,dt$$

spostamento della punta lungo la circonferenza in un intervallo di tempo dt (AL QUADRATO)

$x(t) = \rho(t)\cos\theta(t)$

$y(t) = \rho(t)\sin\theta(t)$

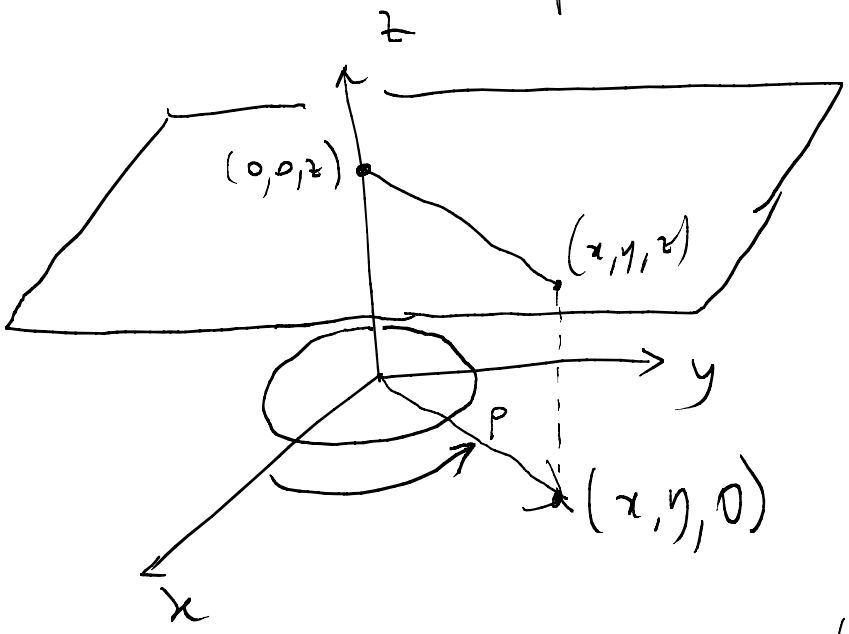
$\dot x \quad \dot y$

$\sqrt{\dot x^2 + \dot y^2}$

$\|\dot\gamma\|$

spostamento in direzione radiale

Coordinate (polari) cilindriche

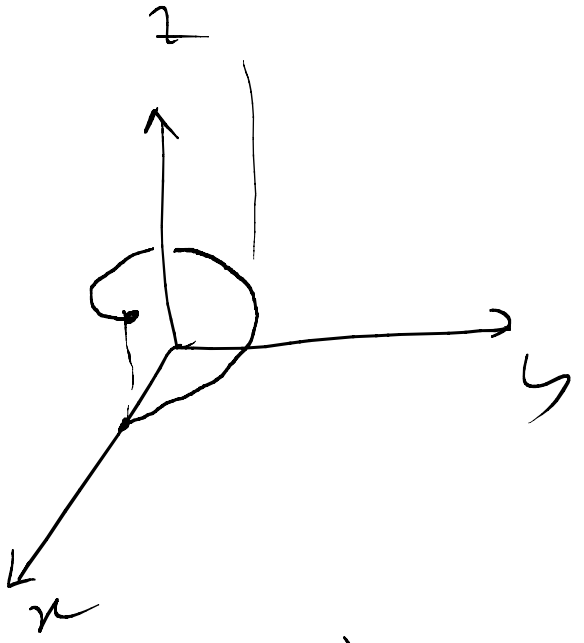


$$\begin{cases} x = \rho \cos\theta \\ y = \rho \sin\theta \\ z = z \end{cases}$$

ρ distanza di (x,y,z) dall'asse z

$$\longrightarrow \quad \gamma(t) = \begin{pmatrix} x(t) \\ y(t) \\ z(t) \end{pmatrix} = \begin{pmatrix} \cos t \\ \sin t \\ t \end{pmatrix}$$

$$\gamma(0) = (1,0,0) \qquad \gamma(2\pi) = (1,0,2\pi)$$



elica (cilindrica)

$\rho(t), \theta(t), z(t)$

$$\begin{cases} x(t) = \rho(t)\cos\theta(t) \\ y(t) = \rho(t)\sin\theta(t) \\ z(t) = z(t) \end{cases}$$

$$\begin{cases} \dot{x}(t) = \dot{\rho}(t)\cos\theta(t) - \rho(t)(\sin\theta(t))\dot{\theta}(t) \\ \dot{y}(t) = \dot{\rho}(t)\sin\theta(t) + \rho(t)\dot{\theta}(t)\cos\theta(t) \\ \dot{z}(t) = \dot{z}(t) \end{cases}$$

$$\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} =$$

$$= \sqrt{\dot{\rho}^2(t) + \rho^2(t)\dot{\theta}^2(t) + \dot{z}^2}$$

$\rho = 1$

$$\begin{cases} x(t) = \cos t & \dot{x} = -\sin t \\ y(t) = \sin t & \dot{y} = \cos t \\ z(t) = t & \dot{z} = 1 \end{cases}$$

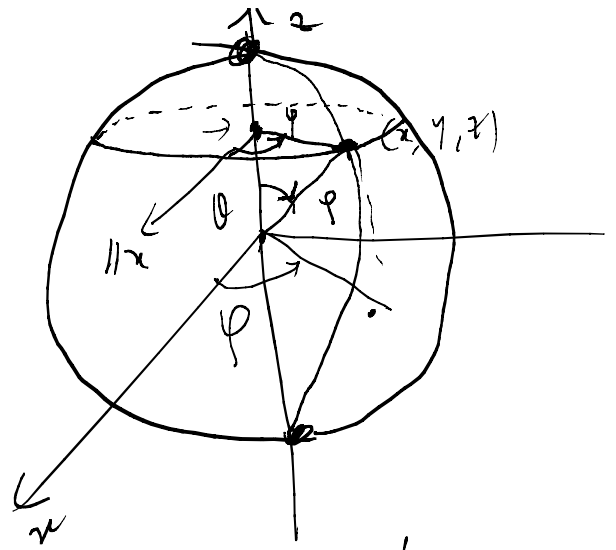
$$\Longrightarrow \int_0^{2\pi} \sqrt{(-\sin t)^2 + (\cos t)^2 + 1}\, dt =$$

$$= \int_0^{2\pi} \sqrt{2}\, dt = 2\pi\sqrt{2}$$

$$\rightarrow x(t) = \rho(t) \sin\theta(t) \cos\varphi(t)$$

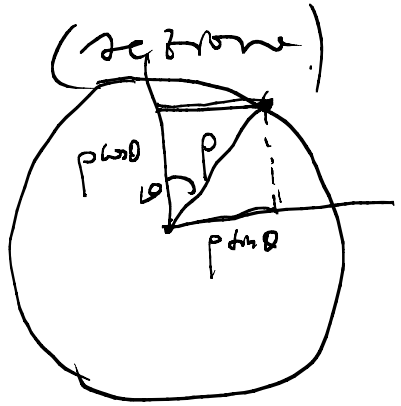
$$\rightarrow y(t) = \rho(t) \sin\theta(t) \sin\varphi(t)$$

$$\rightarrow z(t) = \rho(t) \cos\theta(t)$$



θ = colatitudine
complementare

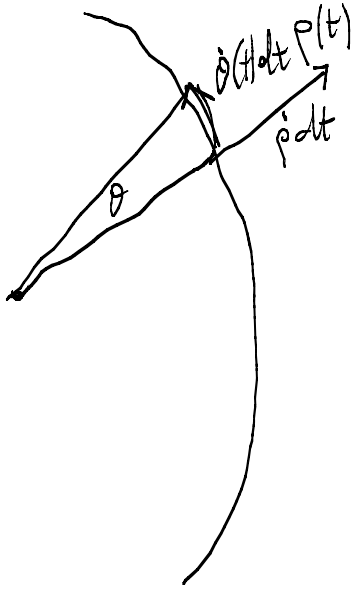
φ è la coordinata angolare della proiezione $(x, y, 0)$ (angolo tra il semiasse positivo di $\bar{x}$ e il raggio vettore di $(x, y, 0)$) (settore)



ρ distanza da $(0,0,0)$

$$\sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2} = \sqrt{\dot{\rho}^2 + \rho^2\dot{\theta}^2 + \rho^2 \sin^2\theta\, \dot{\varphi}^2}$$

- $\dot{\rho}$: spostamento radiale
- $\rho\dot{\theta}$: spostamento Nord-Sud
- $\rho \sin\theta\, \dot{\varphi}$: spost. Est - Ovest



$\dot{\theta}$ = incremento della colatitudine

$\bar{r}$ = il raggio del "parallelo"

$\bar{r} = \rho \sin\theta$

$\dot{\varphi}$ = incremento della longitudine

la formula della lunghezza per le curve in coordinate sferiche è

$$\int_a^b \sqrt{\dot\rho^2(t) + \rho^2(t)\dot\theta^2(t) + \rho^2(t)\sin^2\theta(t)\,\dot\varphi^2(t)}\,dt$$

elemento di linea
(line element) ds, dl

$$dl = \sqrt{\dot x^2 + \dot y^2 + \dot z^2}\,dt$$

cilindriche $= \sqrt{\dot\rho^2 + \rho^2\dot\theta^2 + \dot z^2}\,dt$

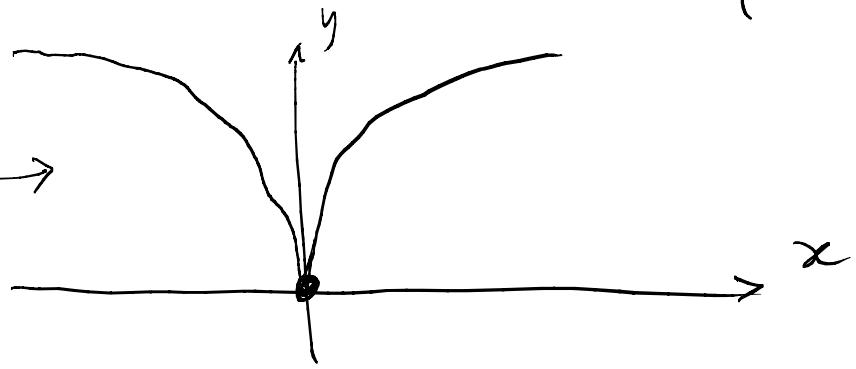
CURVE REGOLARI (E NON)

$\gamma(t) = \begin{pmatrix} t^3 \\ t^2 \end{pmatrix}$ $\quad t \in]-1,1[$ $\quad \boxed{\gamma \in C^\infty\,]-1,1[}$

$x = t^3$
$y = t^2$
eliminare t
Della I eqnaz.

$t = x^{1/3}$ e, sostituendo nella II eq.

Si ottiene $\boxed{y = x^{\frac{2}{3}}}$



$\exists\, y'(0)$?

$y'(x) = \frac{2}{3} x^{-\frac{1}{3}}$

NON HA SENSO per $x=0$

$$\lim_{h\to 0} \frac{y(h)-y(0)}{h} = \lim_{h\to 0} \frac{h^{\frac{2}{3}}-0}{h} =$$

$$= \lim_{h\to 0} h^{-\frac{1}{3}} = \frac{1}{h^{1/3}}$$

NON ESISTE (OSCILLA)

$$\dot\gamma(t) = \begin{pmatrix} 3t^2 \\ 2t \end{pmatrix}$$

$$\dot\gamma(0) = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \gamma \text{ NON È REGOLARE in }]-1,1[$$

$\dot\gamma(t_0) \neq 0 \Rightarrow$ una delle componenti scalari di $\dot\gamma \neq 0$

$\gamma \in C^1 \Rightarrow \dot\gamma \in C^0 \Longrightarrow$ resta diversa da zero in un intervallo di centro t_0.

$[t_0-\delta, t_0+\delta]$ su cui la componente scalare, per esempio la prima $\dot\gamma_1$ è $\neq 0$, per esempio < 0.

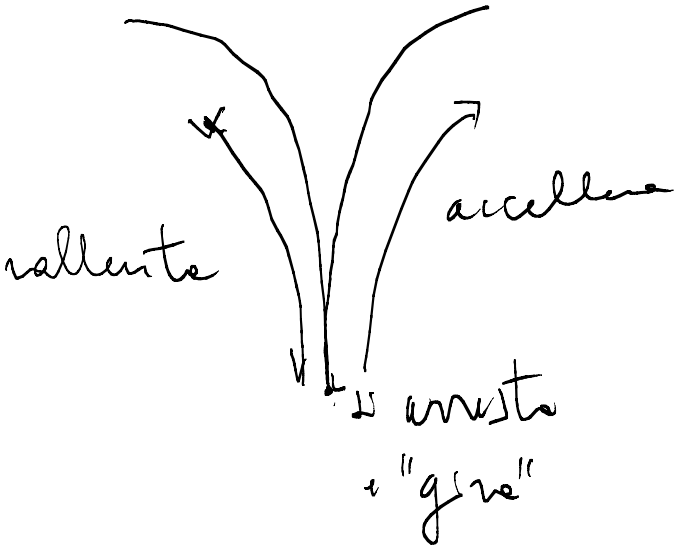
γ_1 è strettamente decrescente su $[t_0-\delta, t_0+\delta]$ $\downarrow$

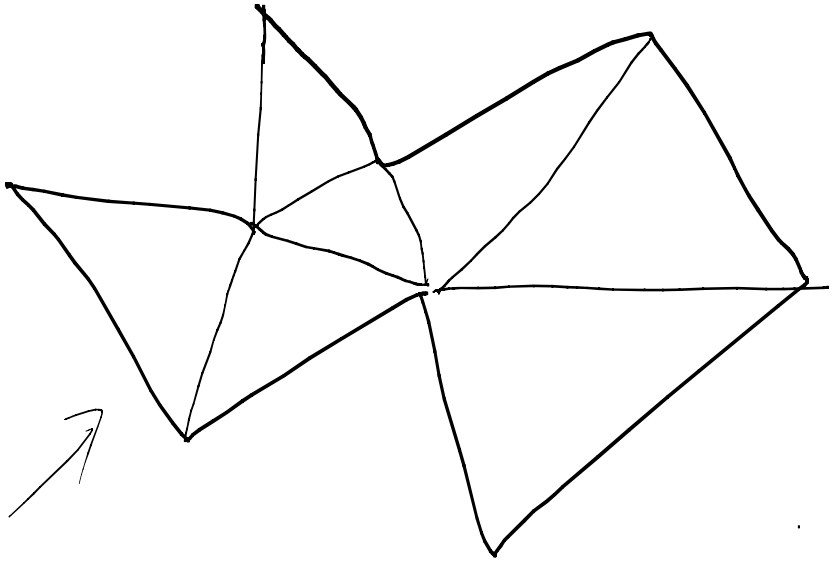
$\Rightarrow$ invertibile

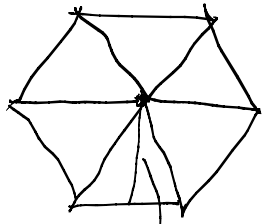
$x_1 = \gamma_1(t) \Rightarrow t = \gamma_1^{-1}(x_1)$

$\gamma_i(t) = \gamma_i(\gamma_1^{-1}(t)) \in C^1$

"moto" (t^3, t^2) $t \in \mathbb{R}$ al variare di t

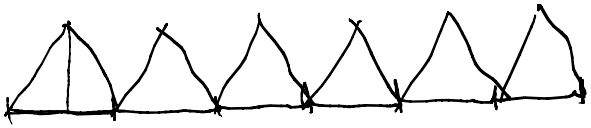




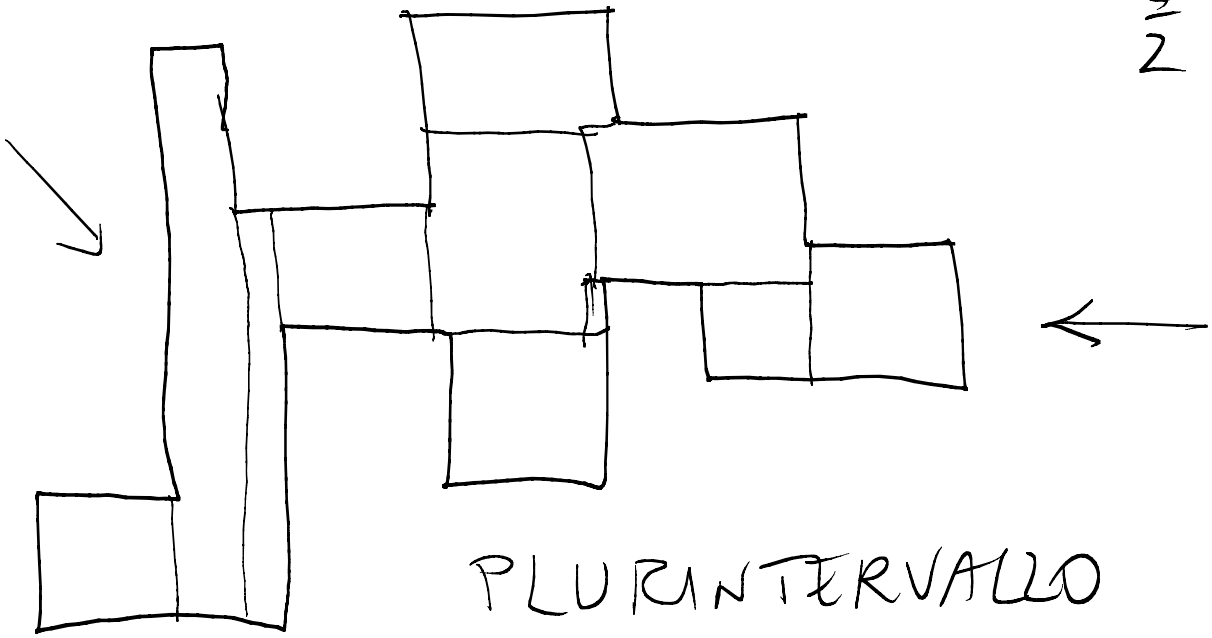


= apotema

=



$\frac{1}{2}$ perimetro x apotema



PLURINTERVALLO