Note Title 5/14/2021

Se $\gamma : [a,b] \to \mathbb{R}^n$ e $\sigma : [c,d] \to \mathbb{R}^n$ sono equivalenti allora $\Lambda(\gamma) = \Lambda(\sigma)$

DIM.

$\exists \rho : [c,d] \to [a,b]$ strettamente monotona

$$\sigma(s) = \gamma(\rho(s)) \qquad \dot\sigma(s) = \dot\gamma(\rho(s))\,\dot\rho(s)$$

$$\Lambda(\sigma) = \int_c^d |\dot\sigma(s)|\,ds = \int_c^d |\dot\gamma(\rho(s))|\,|\dot\rho(s)|\,ds$$

Se γ e σ sono equivalenti equiverse $\dot\rho \geq 0$ e dunque l'ultimo integrale vale

$$\int_c^d |\dot\gamma(\rho(s))|\,\dot\rho(s)\,ds \overset{\substack{t=\rho(s) \\ dt=\dot\rho(s)ds}}{=\!=\!=} \int_{\rho(c)=a}^{\rho(d)=b} |\dot\gamma(t)|\,dt = \Lambda(\gamma)$$

Se γ e σ sono equivalenti antiverse, $\dot\rho \le 0$ e l'integrale viene

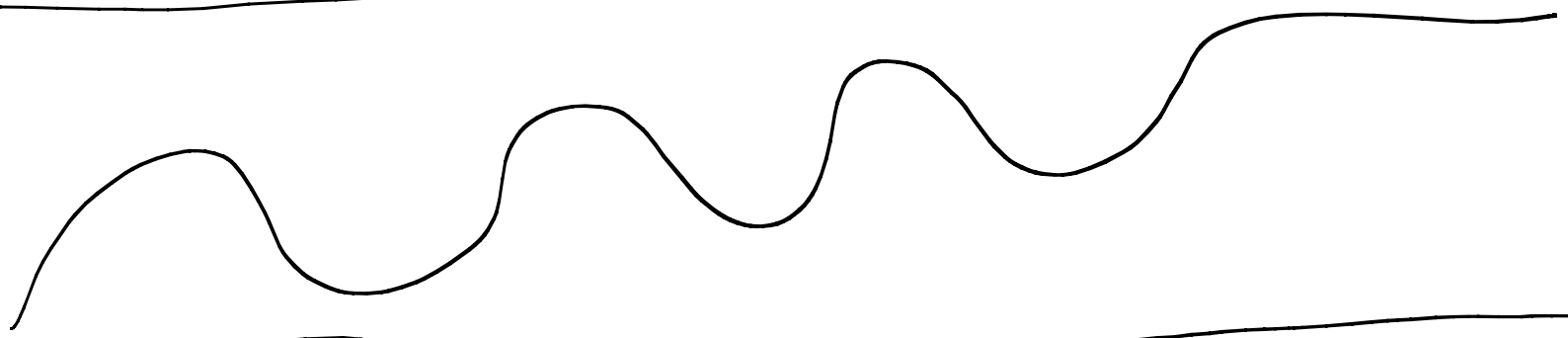
$$\int_c^d |\dot\gamma(\rho(s))|\,(-\dot\rho(s))\,ds = -\int_c^d |\dot\gamma(\rho(s))|\,\dot\rho(s)\,ds =$$

$$\overset{\substack{t=\rho(s)\\ \Rightarrow\\ dt=\dot\rho(s)ds}}{=} -\int_{\rho(c)}^{\rho(d)} |\dot\gamma(t)|\,dt = -\int_b^a |\dot\gamma(t)|\,dt = \Lambda(\gamma)$$

Perché $\rho : [c,d] \to [a,b]$ è strett. decrescente e quindi

$$\rho(c) = b \qquad \rho(d) = a$$

ASCISSA CURVILINEA



$\gamma : [a,b] \to \mathbb{R}^n$ sarà detta **REGOLARE**

se $\dot{\gamma}(t) \neq 0 \quad \forall t \in [a,b]$ e $\gamma \in C^1[a,b]$

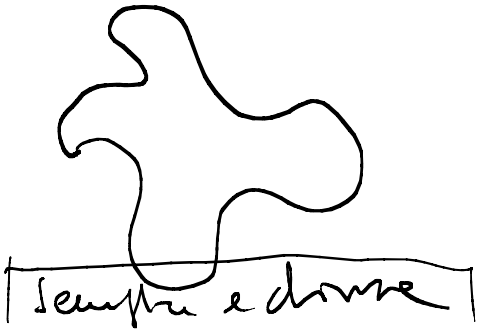
sarà detta **SEMPLICE**

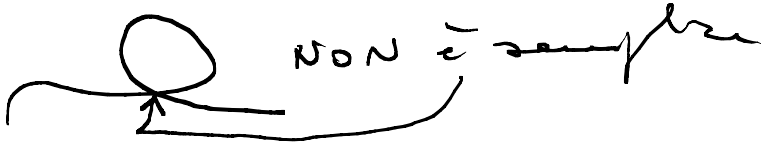
se γ è iniettiva su $[a,b]$. $\gamma(s) = \gamma(t) \Rightarrow s = t$

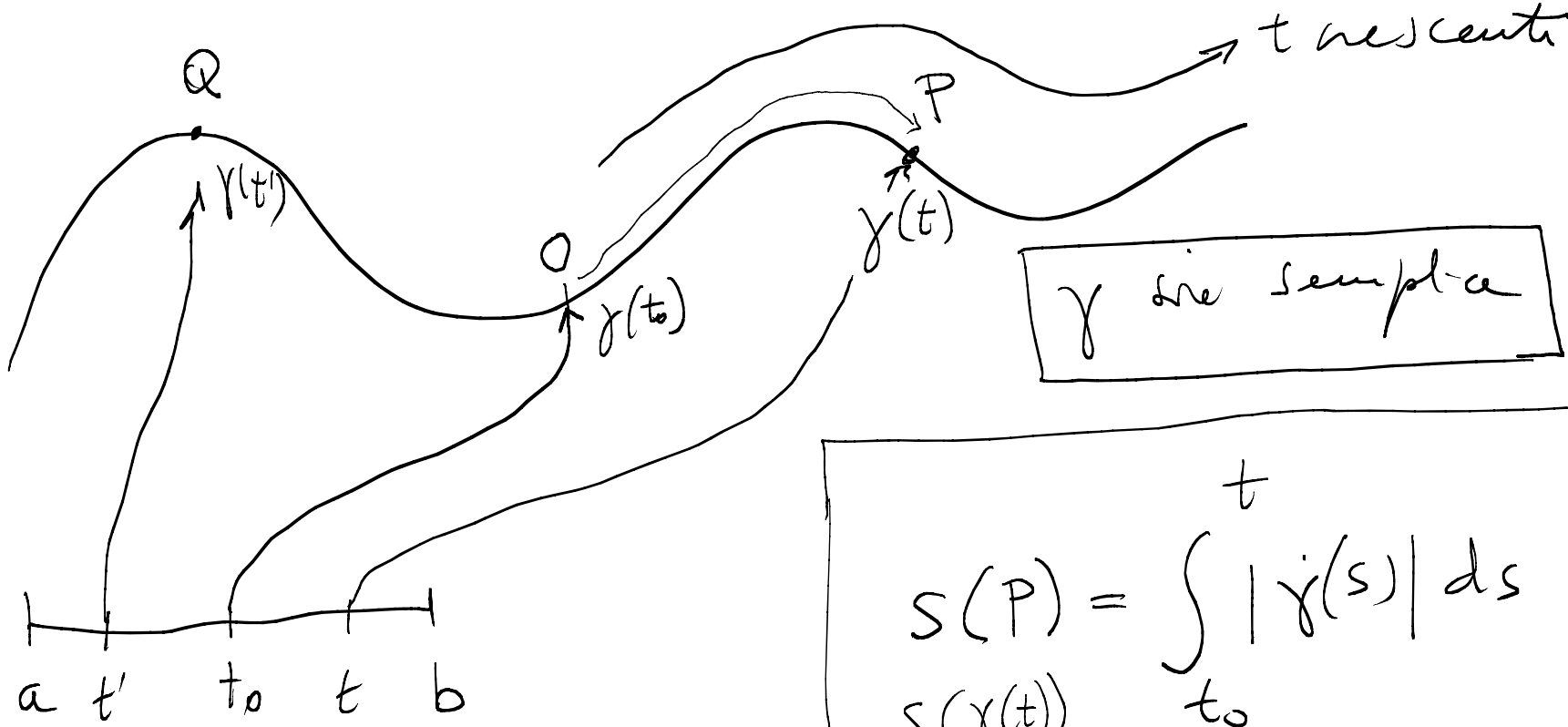
salvo che può accadere che $\gamma(a) = \gamma(b)$

sarà detta **CHIUSA**

se $\gamma(a) = \gamma(b)$







γ sia semplice

$$s(P) = \int_{t_0}^{t} |\dot\gamma(s)|\, ds$$

$s(\gamma(t))$

P segue O, ascissa è positiva

Q precede O, ascissa è negativa

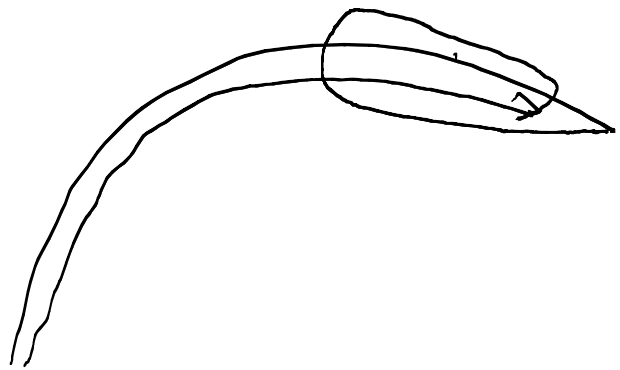
$s(\gamma(t'))$

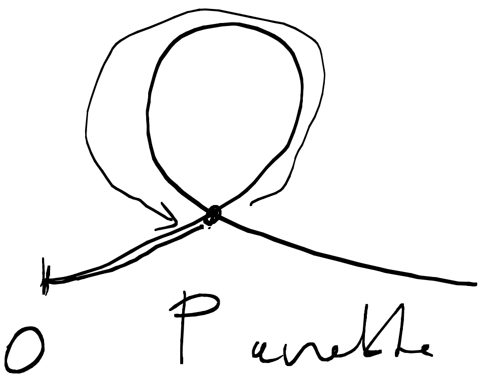
$$s(Q) = \int_{t_0}^{t'} |\dot\gamma(s)|\, ds = -\int_{t'}^{t_0} |\dot\gamma(s)|\, ds \geq 0$$

$$|\dot{\gamma}(t)| \in C^0[a,b]$$

$$\frac{ds}{dt} = |\dot{\gamma}(t)| > 0$$

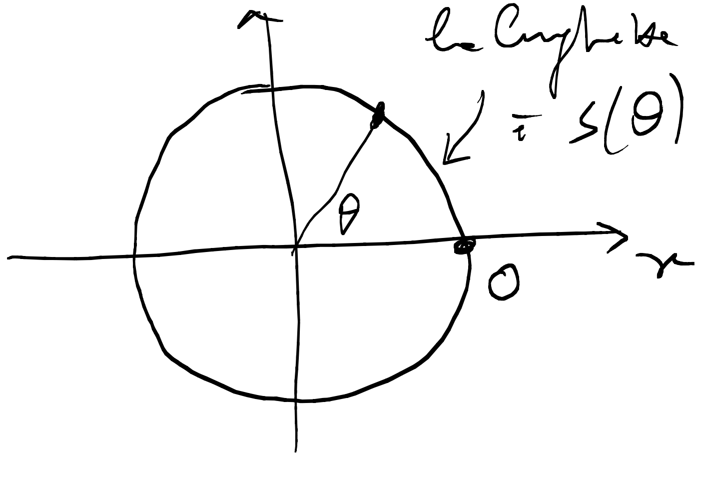
← γ regolare





Punto ha 2 ascisse curvilinee diverse

Se γ è semplice il problema non sussiste.



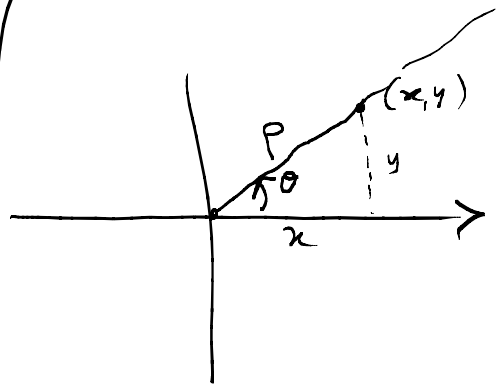
$$\gamma\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} \quad [0, 2\pi]$$

$$\dot{\gamma} = \begin{pmatrix}-\sin\theta \\ \cos\theta\end{pmatrix}$$

$$s(\theta) = \int_0^\theta \sqrt{\sin^2\theta + \cos^2\theta}\, d\theta = \theta$$

$$\gamma(t) = \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} \qquad t \in [a, b]$$

$$\Lambda(\gamma) = \int_a^b \sqrt{\dot{x}^2(t) + \dot{y}^2(t)}\, dt$$



$$\rho = \sqrt{x^2 + y^2}$$

$\rho(t), \theta(t)$ curve parametrizzate in coordinate polari

SPIRALE DI ARCHIMEDE

$$x = \rho \cos\theta$$

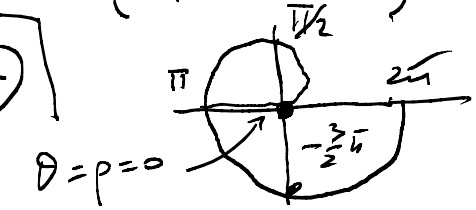
$$y = \rho \sin\theta$$

COORDINATE POLARI (PIANE)

$$x(t) = \rho(t) \cos\theta(t)$$

$$y(t) = \rho(t) \sin\theta(t)$$

$$\dot{x}(t) = \dot{\rho} \cos\theta - \rho \dot{\theta} \sin\theta$$

$$\dot{y} = \dot{\rho} \sin\theta + \rho\dot{\theta} \cos\theta$$

$\rho = \theta$ $\quad [0, 2\pi]$